The sprockets and chain of a bicycle are shown. The pedal sprocket (PS) has a radius of 4 in, the wheel sprocket (WS) has a radius of 2 in, and the wheel has a radius of 13 in. The cyclist pedals at 38 rpm.

(a) Find the rotations per minute (rpm) of the wheel sprocket (WS).
(b) Find the speed of the bicycle. (Assume that the wheel turns at the same rate as the wheel sprocket. Round ans. to 2 decimal places.)

**Solution**

For the PS

1 rev = 1 circumference = $2\pi r = 2\pi \cdot 4$ in = $8\pi$ in.

\[
1 \text{ rpm} = \frac{1 \text{ revolution per min.}}{1 \text{ minute}} = \frac{1 \text{ circumference}}{1 \text{ minute}} = \frac{2\pi r \text{ in}}{1 \text{ min.}} = 8\pi \text{ in/min}.
\]

So 38 rpm = $38 \cdot 8\pi$ in/min. = $304\pi$ in/min.

For the WS: This $304\pi$ in/min. is transferred to the WS, so the linear velocity of the WS is $304\pi$ in/min.

But \[\frac{v}{t} = \frac{r\theta}{t}\] 

So \[\frac{r\theta}{t} = \frac{304\pi}{1 \text{ min}}\]

\[r\theta = 304\pi \text{ in/min}, \text{ or } s = 304\pi \text{ in} \]
Now 1 rev of the WS (and, consequently, 1 rev of the wheel, W) uses \( C = 2\pi r = 2\pi \cdot 2\text{ in} = 4\pi \text{ in} \) of "chain."

So \( \frac{304\pi \text{ in.}}{4\pi \text{ in.}} = 76 \text{ rev. of the WS.} \)

Thus, the wheel sprocket rotates at 76 rpm. Ans to (a).

The wheel also rotates at 76 rpm.

But for the wheel 1 rev = 1 circumference

\[ = 2\pi \cdot 13 \text{ in} = 26\pi \text{ in.} \]

Thus \( \frac{76 \text{ rev}}{\text{min}} = \frac{76 \cdot 26\pi \text{ in}}{\text{min}} \)

And the speed of the bicycle is

\[ V = \frac{s}{t} = \frac{76 \cdot 26\pi \text{ in}}{1 \text{ min}} = 1976\pi \frac{\text{in}}{\text{min}} \]

But we want mph (\( \frac{\text{mi}}{\text{hrs.}} \)).

\[ V = 1976\pi \frac{\text{in}}{\text{min}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{60 \text{ min}}{1 \text{ hr}} \]

\[ = \frac{1976\pi \cdot 60}{12 \cdot 5280} \frac{\text{mi}}{\text{hr}} \approx 5.878586253 \text{ mph.} \]

\[ \approx 5.88 \text{ mph.} \]

The speed of the bicycle is approximately 5.88 mph.