The sine function and the cosine function (and their four close relatives – the cosecant, secant, tangent and cotangent functions) are very important in both applied and theoretical disciplines.

It is extremely important that you be able to understand the graphs of these functions. And “understand” is a big word. I mean that you must be able to draw the graphs and to read (and interpret) the graphs.

**SKILL #1 – DRAWING THE GRAPHS**

A. Let’s start with the basic sine function \( y = \sin x \).

We have already learned that the Period of this function is \( 2\pi \) and its Amplitude is 1. A graph of its fundamental period looks like this:

Now you must understand that I’m creating these images with a TI-83 graphing calculator; therefore, the positive x-axis and the positive y-axis are not labeled as they should be. Also, the scales on each axis are not identified.

In this document, the positive x-axis will always be to the right and the positive y-axis will always be up.

The scales will be identified in the text associated with each graph. In this graph the x-scale is \( \pi/2 \) and the y-scale is 1.

B. We can alter the period of the graph by adjusting the argument of the function. In general, we have

\[
\begin{align*}
  y &= \sin kx \\
  y &= \sin (kx)
\end{align*}
\]

where the “k” controls the period. In our first example above, we have \( k = 1 \). If we take \( k = 2 \), we get

\[
y = \sin (2x)
\]

and using the same window as before, we get:

And as you can see, the period is \( \pi \), but the amplitude is still 1.

Also, notice that my Xmax is just a little more than \( 2\pi \) and that my Ymin and Ymax are just a bit over what I really need. This just makes the graph a little easier to read.
Now let’s try \( k = 3 \). So my function is

\[
y = \sin(3x)
\]

I’m going to take the Xscale to be \( \pi/3 \) this time. I think you’ll see why when you look at the graph.

The “tick-marks” on the x-axis are at \( \pi/3 \) intervals, and they are being covered up by the graph. So it takes 2 of these tick-marks to make a period. So the period of this function is \( 2\pi/3 \).

Another way to see this is to observe that there are three periods in this graph. So each period must be one-third of the \( 2\pi \) interval. So each period is \( 2\pi/3 \).

Thus, if I tell you that, given the function \( y = \sin(kx) \), the period is given by the formula

\[
P = \frac{2\pi}{k}
\]

you may have some reason to believe me.

The same formula for period holds true for the corresponding cosine function, \( y = \cos(kx) \).

C. The amplitude of the basic sine function \( y = \sin x \) is 1. That is, its graph goes up to 1 vertically and down to -1. But what if I look at \( y = 2\sin x \)?

Here you may notice that I changed both scales in my “Window.” The x-scale is \( \pi/6 \). I really didn’t have to do this; I did just to let you see a different set of “tick-marks” on the x-axis.

However, you do need to pay close attention to the scale on the y-axis. The Yscl=0.5. So you see that the graph goes “up” to 2 and “down” to -2.

The period is still the basic \( 2\pi \) (\( \frac{12\pi}{6} = 2\pi \)), but the amplitude is 2.
So it should be easy for you to accept the formula for amplitude. If you are given the equation \[ y = a \sin x \], the amplitude is \[ A = |a| \] where A stands for amplitude. Notice that the amplitude is the absolute value of the leading coefficient.

D. So now if you see a function, say for example \[ f(x) = -\frac{1}{2} \cos(4x) \], you should immediately know that it is a cosine function, its period is \[ P = \frac{2\pi}{k} = \frac{2\pi}{4} = \frac{\pi}{2} \] and its amplitude is \[ A = |a| = \frac{1}{2} \]. Thus, before you start graphing, you know that the graph completes one cycle in π/2 units, and it goes up to 1/2 and down to -1/2. But, because of the “minus sign,” it goes down first!

Do you see that we’ve got an “upside-down” cosine function here? It “starts out” at -0.5 rather than at 0.5. And it goes up.

Also note that its amplitude is 0.5.

Also, since I set the Xscl at π/4, the graph shows that the value of the function is zero at π/8 (half way between 0 and π/4), 0.5 at π/4, 0 at 3π/8, -0.5 at π/2, and so on. The graph completes its entire cycle in the interval \( 0, \frac{\pi}{2} \). Thus, the graph above shows 4 periods (cycles) of the graph in the interval \( 0, 2\pi \).

I put in these two “screen-shots” just to convince you that the first zero of the graph occurs at π/8, as I have been claiming.

Here you get the added benefit of seeing that \( \frac{\pi}{8} \approx 0.39269908 \), not that there is any reason that you should have to memorize this. Goodness knows that we have enough other stuff to memorize in trig.

E. Now, the main point that I’m trying to make in this paper is that you can get a pretty good idea of what the graph of a sine or cosine function looks like if you can plot the values of the function for five points on the x-axis – namely, the beginning point (of a period), the ending point of that same period, the “1/4 point,” the “1/2 point,” and the “3/4 point.”
For example, suppose that a period starts at $0$ and ends at $2\pi$ (and we’ve already seen a bunch of these). Let’s just take this example: 

$$f(x) = 1.5 \cos(x).$$

Then the length of the period is 

$$\text{end} - \text{beginning} = \text{length} = 2\pi - 0 = 2\pi$$

and so it follows that the three intermediate points are

Thus, the **five points** are \(\left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \right\}.\)

This means that the function achieves its max values, its min values, and its zeros at these five points. And I know that its max value is 1.5 and its min value is -1.5 and its zero values are 0. I just have to get it straight which are which, plot the points, and draw a **reasonable curve** connecting the points.

F. Examples: Graph each of the following using the **five point method**.

1. \(f(x) = 2 \sin(5x)\)

   **Solution:**
   
   a. Period: \(P = \frac{2\pi}{k} \text{ and } k = 5; \text{ therefore, } P = \frac{2\pi}{5}.\)
   
   b. Amplitude: \(A = a \text{ and } a = 2; \text{ therefore, } A = 2.\)
c. 5 points: (i). The “argument” of the function must go from 0 to $2\pi$. In this case, the argument of the function is $5x$. Thus, the period begins at $\begin{equation} 5x = 0 \end{equation}$ and it ends at $\begin{equation} x = 0 \end{equation}$. And so the length of the period is $\begin{equation} \frac{2\pi}{5} - 0 = \frac{2\pi}{5} \end{equation}$. I think that we already knew this, but here are the formats for finding the beginning and end points in more complicated cases – which we shall get to in the future.

(ii). Now for the three intermediate points:

\[
\begin{align*}
\text{1/4 point} & = \frac{1}{4} \times \frac{2\pi}{5} = \frac{\pi}{10} \\
\text{1/2 point} & = \frac{1}{2} \times \frac{2\pi}{5} = \frac{\pi}{5} \\
\text{3/4 point} & = \frac{3}{4} \times \frac{2\pi}{5} = \frac{3\pi}{10}
\end{align*}
\]

(iii). So our 5-points are

\[
\left\{ 0, \frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10}, \frac{4\pi}{10} \right\} = \left\{ 0, \frac{\pi}{5}, \frac{\pi}{5}, \frac{3\pi}{10}, \frac{2\pi}{5} \right\} \hspace{1em} \text{in reduced form.}
\]

d. The values of the function at the 5-points are:

\[
\begin{array}{c|c|c}
X & Y_1 \\
n & \text{value} & \text{value} \\
0 & 0 & 0 \\
.31416 & .62832 & 0 \\
.62832 & .95493 & -2 \times 10^{-13} \\
.95493 & 1.2566 & -2 \times 10^{-13} \\
1.2566 & 1.5708 & -2 \times 10^{-13} \\
1.5708 & 1.885 & \end{array}
\]

Look just at the first 5 x’s. These are the decimal approximations of the 5-points. Thus, the first 5 y’s give the y-coordinates of our graph. So I expected 0’s and ±2’s, but what about the −2E−13? Well, you may recall that $-2E-13 = -2 \times 10^{-13} = -0.00000000000002 \approx 0$. Remember, the calculator usually approximates when its output is a long string of digits. So the $-2E-13$ is just the calculator’s way of saying zero in this particular calculation.

Thus to graph this function using the 5-point method, you

1. Mark the 5 points on the x-axis.
2. Plot the ordered pairs, which are above, on, or below the x-axis
3. Draw a smooth “trig curve” connecting the points.

We’ll have to do a few of these in class for you to see exactly what to do.

Here’s what it looks like on a “2π-window.”
And here's what it looks like in your TI-83 or TI-84 graphing window.

And here's what your P&P (pencil and paper) work might look like:

\[ f(\theta) = 2 \sin(5\theta) \]

**Graph**

Work:

1. \( f(0) = 2 \sin(0) = 2 \times 0 = 0 \)
2. \( f\left(\frac{\pi}{10}\right) = 2 \sin\left(\frac{\pi}{2}\right) = 2 \times 1 = 2 \)
3. \( f\left(\frac{\pi}{5}\right) = 2 \sin\left(\frac{\pi}{2}\right) = 2 \times 1 = 2 \)
4. \( f\left(\frac{\pi}{3}\right) = 2 \sin\left(\frac{\pi}{2}\right) = 2 \times 1 = 2 \)
5. \( f\left(\frac{\pi}{2}\right) = 2 \sin\left(\frac{\pi}{2}\right) = 2 \times 1 = 2 \)

These points can be plotted on the graph to show the behavior of the function.

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**EXTRA INFORMATION**

- **WINDOW**
  - \( X_{\text{min}} = 0 \)
  - \( X_{\text{max}} = 2\pi \)
  - \( Y_{\text{min}} = -2 \)
  - \( Y_{\text{max}} = 2 \)
  - \( X_{\text{scale}} = \frac{\pi}{10} \)
  - \( Y_{\text{scale}} = 1 \)

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**REFERENCES**

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