Example — How To Do “Opposite Things In Opposite Order" to find inverses to Quadratic Functions.

Of course quadratic functions don't have inverses (their graphs are parabolas & parabolas don't pass the "horizontal line test" — they are NOT one-to-one functions). However, if we restrict the domain of a parabola (quadratic function), we can take one side of the parabola and we've got a one-to-one function and this will make it possible to have an inverse.

For example, if we have \( f(x) = x^2 - 2x + 2 \), its domain is \((-\infty, \infty)\). It is not 1-1 (one-to-one) on its domain; however, if we restrict the domain to \([-1, \infty)\), we are looking at the "right half" of the parabola, and THAT is 1-1.

Now to calculate the inverse, I have to re-structure \( f \) as follows:

\[
f(x) = x^2 - 2x + 2 = x^2 - 2x + 1 + 1 = (x^2 - 2x + 1) + 1 = (x-1)^2 + 1
\]

So \( f(x) = (x-1)^2 + 1 \). You may have see this done before.

Anyway, now I have \( f \) expressed as a sequence of operations applied to the "input value" \( x \).

\( f \) says: Take \( x \), Subtract 1. Square the result. Add one to that.

Thus here is \( f^{-1} \) in steps.

\[
x \rightarrow x - 1 \rightarrow \sqrt{x-1} + 1
\]

\[
f^{-1}(x) = \sqrt{x-1} + 1 \quad \text{if } x \geq 1
\]
That one might have been too easy. Try this:

\[ f(x) = x^2 + 6x + 8 \]

Find an inverse with restriction.

**Solution:**

1. You might be tempted to factor this: \( f(x) = (x+2)(x+4) \), but this does no good whatsoever.

2. Here's the set-up:

\[ f(x) = x^2 + 6x + 8 = x^2 + 6x + 9 - 1 = (x + 3)^2 - 1 \]

\[ \therefore \text{ if } x \geq -3 \quad \text{(Vertex of parabola is } \sqrt{-3}, -1) \]

\[ f^{-1}(x) = \sqrt{x+1} - 3 \]