**Example 4: Square Roots of Negative Numbers**

(a) $\sqrt{-1} = i$  
(b) $\sqrt{-16} = 4i$  
(c) $\sqrt{-3} = i\sqrt{3}

We usually write $i\sqrt{b}$ instead of $\sqrt{bi}$ to avoid confusion with $\sqrt{b}$.

**Example 5: Using Square Roots of Negative Numbers**

Evaluate $(\sqrt{12} - \sqrt{-3})(3 + \sqrt{4})$ and express in the form $a + bi$.

**Solution**

$(\sqrt{12} - \sqrt{-3})(3 + \sqrt{4}) = (\sqrt{12} - i\sqrt{3})(3 + 2i) = (2\sqrt{3} - i\sqrt{3})(3 + 2i)

= (6\sqrt{3} + 2\sqrt{3}i + 6\sqrt{3}i + 3\sqrt{3})

= 8\sqrt{3} + 8\sqrt{3}i

So, the solutions are $-2 + i$ and $-2 - i$.

**Example 6: Quadratic Equations with Complex Solutions**

Solve each equation.

(a) $x^2 + 9 = 0$  
(b) $x^2 + 4x + 5 = 0$

**Solution**

(a) The equation $x^2 + 9 = 0$ means $x^2 = -9$, so

$x = \pm 3\sqrt{-1}$

The solutions are therefore $3i$ and $-3i$.

(b) By the quadratic formula we have

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For $x^2 + 4x + 5 = 0$, $a = 1$, $b = 4$, and $c = 5$.

$x = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm 2i}{2}$

$= -2 \pm i$

So, the solutions are $-2 + i$ and $-2 - i$.

**Example 7: Complex Conjugates as Solutions of a Quadratic**

Show that the solutions of the equation

$4x^2 - 24x + 37 = 0$

are complex conjugates of each other.

**Solution**

We use the quadratic formula to get

$x = \frac{24 \pm \sqrt{24^2 - 4(4)(37)}}{2(4)}$

$= \frac{24 \pm \sqrt{576 - 4(37)}}{8}$

$= \frac{24 \pm \sqrt{36}}{8}$

$= \frac{24 \pm 6}{8}$

$= -\frac{3}{2}$ and $\frac{1}{2}$, which are complex conjugates of each other.

So, the solutions are $3 + \frac{1}{2}$ and $3 - \frac{1}{2}$, and these are complex conjugates.

**Exercises**

1. Find the real and imaginary parts of the complex number.

(a) $5i$  
(b) $-6 + 4i$

2. $\frac{4}{2}$  
3. $6 - i$

4. $\sqrt{3}$  
5. $\sqrt{8}$

6. $\sqrt{3} + \sqrt{-3}$  
7. $2 - \sqrt{-3}$

Find the sum of the following complex numbers.

8. $(2 - 5i) + (3 + 4i)$

9. $(2 + 5i) + (4 - 6i)$

10. $(-6 + 6i) + (9 - i)$

11. $(3 - 2i) + (-5 + i)$

12. $(6 - 4i)$

13. $(5 + 6i)$

14. $(4 - 3i)$

15. $(8 + 2i)$