1. Given the sequence 1, 3, 7, 15, 31, …, find [A] \( a_6 \) and [B] the general term, \( a_n \).

2. Suppose you have an AS (arithmetic sequence) with \( a_4 = 19 \) and \( a_6 = 27 \). Find [A] the first term, \( a_1 \), [B] the 12-th term, \( a_{12} \) and [C] the n-th term, \( a_n \) (and simplify it). – SHOW WORK.

3. List the first 5 terms of the sequence defined recursively by \( a_n = 4(a_{n-1} - 2) \), \( a_1 = 4 \) – SHOW WORK.

4. A large outdoor amphitheater has 30 seats in the first row, 36 seats in the second row, 42 seats in the third row, and so on. There are 26 rows altogether. How many seats are in the amphitheater? (Final answer is a sentence.) – SHOW WORK.

5. A geometric sequence has a first term of 8 and a common ratio of \( \frac{2}{3} \). Find the fifth term and state your answer as an improper fraction, not a decimal number. – SHOW WORK.

6. Find the fourth term of the expansion of \((x + 2)^{14}\) – SHOW WORK.

7. Classify each of the following as a (a) statement, (b) open sentence, (c) neither:
   a. Click it or Ticket!
   b. The Sun is twice as big as the Moon.
   c. She is the smartest person in the class.
   d. If you don’t eat your meat, you can’t have any pudding.

8. Translate each of the following into symbolic form using the letters given as affirmative statements.
   a. The apple is red and either the pear is brown or the grapefruit is not yellow. \((r, b, y)\)
   b. If you lock the door and shut all the windows, then the wolf will go away.
      \((l: you lock the door, s: you shut all the windows, g: the wolf will go away)\)

9. Find the sum. \( \sum_{k=1}^{10} 5\left(\frac{1}{2}\right)^{k-1} \) \[Hint: \text{The sequence involved is geometric!}\] – SHOW WORK.

10. Find the sum of the infinite geometric sequence: \( \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \frac{2}{243} + \ldots \) – SHOW WORK.

11. (Bonus). The fifth term of a geometric sequence is \( \frac{243}{32} \) and the common ratio is \( \frac{3}{2} \). What is the third term of the sequence? – SHOW WORK.
#1) Seq. 1, 3, 7, 15, 31, ...  Required: [A] \( a_8 \) and 
[B] \( a_n \).

**Solution:** Notice what the seq. looks like - Add 1 to each term and what do you get?

\[ 2, 4, 8, 16, 32, ... \]
\[ 2^1, 2^2, 2^3, 2^4, 2^5, ... \]

So the seq. is \( 2^1 - 1, 2^2 - 1, 2^3 - 1, ... \)

So [A] \( a_8 = 2^8 - 1 = 256 - 1 = 255 \) and

[B] \( a_n = 2^n - 1 \)

#2) AS \( a_4 = 19 \) \( a_6 = 27 \)  Required: [A] \( a_1 \) [B] \( a_{12} \), 
[C] \( a_n \) simplify (Show work).

**Solution:** [1] \( a_n = a + (n-1)d \). [2] \( 27 = a_6 = a + 5d \)
\[ 19 = a_4 = a + 3d \]

[3] \( a + 5d = 27 \)
\[-a - 3d = -19 \]
\[ 2d = 8 \] \( d = 4 \) \( a = 3 \)

[4] \( a + 5d = 27 \)
\[ a + 20 = 27 \]
\[ a = 7 \]

[5] \( a_1 = 7 \) \( A \), [C] \( a_{12} = a + 11d = 7 + 11 \cdot 4 = 51 \) \( a_{12} = 51 \)

[7] \( a_n = a + (n-1)d = 7 + (n-1)4 = 7 + 4n - 4 = 4n + 3 \)
\[ a_n = 4n + 3 \] \( C \)

#3) 5 terms: \( a_n = 4(a_{n-1} - 2) \), \( a_1 = 4 \) (Show Work)

**Solution:** \( a_2 = 4(4-2) = 8 \), \( a_3 = 4 \cdot 6 = 24 \), \( a_4 = 4 \cdot 22 = 88 \), \( a_5 = 4 \cdot 86 = 344 \)

[7] \[ 4, 8, 24, 88, 344 \]
1st row: 30 seats; 2nd: 36 seats; 3rd: 42 seats; etc. 
Show Work.
\[ S_n = \frac{n}{2} [2a + (n-1)d] \]
So 
\[ S_{26} = \frac{26}{2} [2 \cdot 30 + (26-1) \cdot 6] = 13 [60 + 25 \cdot 6] = 13 [60 + 150] = 13 [210] = 2730 \]

5. GS. \( a = 8 \), \( r = \frac{2}{3} \). Find \( a_5 \). Fraction. Show work.
Soln 1 \[ a_n = ar^{n-1} \]
\[ a_5 = 8 \left( \frac{2}{3} \right)^4 = 8 \cdot \frac{16}{81} = \frac{128}{81} \]

6. 4th Term of expansion of \((x+2)^{14}\). (Show Work).
Soln 1 \[ \left( \frac{14}{3} \right)x^{11}(2)^3 \] is the 4th term.
\[ \left( \frac{14}{3} \right) \cdot 2 = \frac{2912}{3} \cdot \frac{1}{2} = \frac{26}{3} \]

7. \( a - b \); \( b - a \); \( c - b \); \( d - a \)

8. \[ a: \neg (b \lor \neg y) \quad b: (q \land s) \lor \neg q \]

9. \( \sum_{k=1}^{10} 5 \left( \frac{1}{2} \right)^{k-1} \) (Show Work)
Soln \[ \sum_{k=1}^{10} 5 \left( \frac{1}{2} \right)^{k-1} = 5 \left( \frac{1}{2} \right)^0 + 5 \left( \frac{1}{2} \right)^1 + 5 \left( \frac{1}{2} \right)^2 + \ldots + 5 \left( \frac{1}{2} \right)^9 \]
which I recognize as a geom. series w/ \( a = 5 \), \( r = \frac{1}{2} \), \( n = 10 \).
\[ \text{So } \sum_{k=1}^{10} 5 \left( \frac{1}{2} \right)^{k-1} = S_{10} \quad \text{and} \quad S_n = a \frac{1-r^n}{1-r} \]
\[ S_{10} = 5 \cdot \frac{1-(1/2)^{10}}{1-1/2} = 5 \cdot \frac{1-(1/2)^{10}}{1/2} = 10 \left( 1 - \frac{1}{1024} \right) = 10 \left( \frac{1023}{1024} \right) = \frac{10230}{1024} = \frac{5115}{512} \]
#10 Find sum GS: $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

Soln. \[ a = 2, \quad r = \frac{1}{3}. \]

\[ S_\infty = \frac{a}{1 - r} = \frac{2}{1 - \frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2} = 3 \]

#11 (Bonus) \[ a_5 = \frac{243}{32}, \quad r = \frac{3}{2}. \] GS. Find $a_3$.

Soln. \[ a_4 = \frac{a_5}{r} = \frac{243/32}{3/2} = \frac{243}{32} \times \frac{2}{3} = \frac{81}{16} \]

\[ a_3 = \frac{a_4}{r} = \frac{81}{16} \times \frac{3}{2} = \frac{27}{8} \]

\[
\begin{array}{c}
\text{END}
\end{array}
\]