INSTRUCTIONS:
Do all your work and put your answers (boxed or circled) on the worksheets. Do no more than 3 problems per page on your worksheets. Draw a horizontal line segment between problems. Write only on the front of each worksheet. Do not write in the margins (top, left, and bottom-right) except to number your pages in the page-numbering circle.

1. (12 pts.) The Klingons are attacking my small outpost on Planet XYZ. The first wave (group) has 100 Klingons, the second wave has 122 Klingons, the third wave has 144, and so forth. If 250 waves of Klingons attack, how many Klingons in total must I defend against?

2. (13 pts.) Here is an equation of an ellipse: \[25x^2 + 16y^2 = 400\]
   Required:
   a. Put the equation into our New Standard Form (NSF).
   b. State the coordinates of the vertices and the foci.
   c. State the length of the major axis, the eccentricity, and whether it is vertical or horizontal.

3. (12 pts.) A parabola with vertex at the origin has its focus at \(F(-4,0)\).
   What is its equation in NSF? Also, does it open upward, downward, right, or left?

4. (13 pts.) Solve using CRAMER’s RULE:
   \[
   \begin{aligned}
   2x + 5y &= 16 \\
   8x - y &= 1
   \end{aligned}
   \]
   [If you solve using any other method, you’ll get absolutely no credit. I need to see whether you have learned CRAMER’s RULE.]

5. (12 pts.) A parabola has the equation (in NSF) \(x^2 = 10y\)
   Required:
   a. Coordinates of the focus.
   b. Equation of the directrix.
   c. In which direction does it open?

6. (13 pts.) Solve this system of equations
   \[
   \begin{aligned}
   x^2 + y^2 &= 100 \\
   3x - y &= 10
   \end{aligned}
   \]
   [Hint: I suggest the substitution method.]

7. (12 pts.) Here is a system of linear equations and the associated augmented matrix. Solve using your calculator (if you wish). Show the Reduced Row Echelon Matrix (rref) and then write the solution to the system.
   \[
   \begin{aligned}
   2x + y - z &= 6 \\
   x - 4y + 3z &= -14 \\
   9x + 2y + 5z &= 10
   \end{aligned}
   \]
8. (13 pts.) Find the Partial Fraction Decomposition (PFD) of \( \frac{2x+2}{x^2-6x+9} \).

9. (BONUS – 10 points) Find an equation (in New Standard Form) for the ellipse that satisfies the given condition(s).

Eccentricity \( \frac{2}{3} \) and foci \( F_1(-8,0) \) and \( F_2(8,0) \).
1. Klingon attack - 1st wave 100, 2nd wave 122, 3rd wave 144. Etc. 250 waves. How many Klingons?

Solution:
1. A.S. - need a(n, d), \( a = 100, \ n = 250, \ d = 22 \).
2. Need SUM, \( S_n = \frac{n}{2} [2a + (n-1)d] \).
3. Subs., \( S_{250} = \frac{250}{2} [2(100) + 249 \cdot 22] \).
   \[ = 250 \left[ \frac{2 \cdot 100 + 249 \cdot 22}{2} \right] = 250 \left[ \frac{2(100 + 249 \cdot 11)}{2} \right] \]
   \[ = 250 \left[ 100 + 2739 \right] = 250 \cdot 2839 = 709750. \]
4. Sentence: I must defend against 709750 Klingons.

2. Ellipses - \( 25x^2 + 16y^2 = 400 \), [a] NSF, [b] V \( F_1 \), [c] major x, e_1 Vorh.

Solution:
1. NSF \( \frac{25}{400} x^2 + \frac{16}{400} y^2 = \frac{400}{400}; \frac{x^2}{16} + \frac{y^2}{25} = 1 \).
2. \( a^2 = 25, b^2 = 16; \ a^2 = b^2 + c^2 \) - \( c^2 = a^2 - b^2 = 25 - 16 = 9 \).
   So \( a = 5, b = 4, c = 3 \).
3. \( V_1 (0, -5), V_2 (0, 5), F_1 (0, -3), F_2 (0, 3), \) [b]
4. major = 2a = 10. \( e = \frac{c}{a} = \frac{3}{5} \). Ellipse is Vertical. \( \left\langle [c] \right\rangle \)

3. Parabola. \( V(0, 0) \) \( \frac{1}{2} \) F(-4, 0), [a] Eq, NSF [b] opens?

Solution:
1. F is on neg. x-axis, so parabola opens left. \( \left\langle [b] \right\rangle \)
2. \( p = 4 \), so \( 4p = -16 \). So NSF: \( y^2 = -16x \).

4. Solve W/ CRAMER, - No Cramer, No CREDIT. \( 2x + 5y = 16 \)
   \( 8x - y = 1 \)

Solution:
1. \( D = \frac{2}{5} \left\langle \begin{array}{c}
-2
-40
\end{array} \right\rangle = -42 \)
2. \( \frac{x}{D} = \frac{-2}{42} = \frac{1}{21} \)
3. \( \frac{y}{D} = \frac{-126}{42} = -3 \)
4. Solution: \( \left( \frac{1}{2}, 3 \right) \)
#3 Parabola \( x^2 = 10y \): [a] F [b] dir. [c] open?

**Solution**

1. \( 4p = 10 \), \( p = \frac{10}{4} = \frac{5}{2} \)
2. \( [a] \Rightarrow F(0, \frac{5}{2}) \) \( [b] \Rightarrow y = -\frac{5}{2} \)
3. \( [c] \Rightarrow \) Opens Upward.

#6 Solve \( (s I f s ?) \) \[
\begin{align*}
\frac{x^2 + y^2}{100} & = 1 \quad \text{and} \quad 3x - y = 10
\end{align*}
\]

**Solution**

\[
\begin{align*}
3x & - y = 10 \\
2x^2 + (3x - 10)^2 & = 100 \\
\Rightarrow 10x^2 - 60x & = 0 \\
\Rightarrow x(x - 6) & = 0 \\
\Rightarrow x & = 0 \lor x = 6 \\
[\{0 = 0\} \land (y = 3x - 10)] & \Rightarrow y = -10 \\
[\{x = 6\} \land (y = 3x - 10)] & \Rightarrow y = 8
\end{align*}
\]

The solution set is \( \{(0, -10), (6, 8)\} \).

#7 Solve on Calculator.

\[
\begin{align*}
2x + y & - z = 6 \\
\text{and} \quad x - 4y + 3z & = -14 \\
\text{and} \quad 9x + 2y + 5z & = 10
\end{align*}
\]

**Solution**

\[
\begin{bmatrix}
2 & 1 & -1 \\
1 & -4 & 3 \\
9 & 2 & 5
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
6 \\
-14 \\
10
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

#8 PFD \( \frac{2x^2 + 2}{x^2 - 6x + 9} \)

**Solution**

\[
\begin{align*}
\frac{2x^2 + 2}{(x-3)^2} & = \frac{A}{x-3} + \frac{B}{(x-3)^2} \\
\Rightarrow 2x + 2 & = A(x-3) + B \\
\Rightarrow 2x + 2 & = x - 3A + B \\
\Rightarrow 2x + 2 & = \frac{2x + 2}{x^2 - 6x + 9}
\end{align*}
\]

\( A = 2 \)

#9 Bonus Ellipse NSF s.t. \( e = \frac{2}{3} \) foci \( F_1(-8,0), F_2(8,0) \).

**Solution**

1. \( C = 8 \), ellipse is horizontal
2. \( \frac{2}{3} = e = \frac{c}{a} = \frac{8}{a} \Rightarrow a = 12 \)
3. \( a^2 = b^2 + c^2, \quad 144 = b^2 + 64 \Rightarrow b^2 = 80 \)
4. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (horiz.)

\[
\begin{bmatrix}
\frac{x^2}{144} + \frac{y^2}{80} = 1
\end{bmatrix}
\]

End