#1/10 (A) Eval. Exact

(a) \( \log_4 (\sqrt{2}) = \log_4 (2^{1/2}) = x \) so \( 4^x = 2^{1/2} \)

So \( 2^{2x} = 2^{1/2} \) so \( 2x = \frac{1}{2} \) so \( x = \frac{1}{4} \)

\[ \log_4 (\sqrt{2}) = \frac{1}{4} \]

(b) \( \log_4 (8) = x \) so \( 4^x = 8 \) so \( 2^{2x} = 2^3 \)

So \( 2x = 3 \), so \( x = \frac{3}{2} \)

\[ \log_4 (8) = \frac{3}{2} \]

(B) Use defn:

(a) \( \log_3 (25) = 2 \) so \( x^2 = 25 \)

\[ x = 5 \]

(b) \( \log_3 (8) = \frac{2}{3} \) so \( x^{2/3} = 8 \)

\[ (x^{2/3})^{3/2} = (8)^{3/2} = 8 \cdot 8^{1/2} = 8 \cdot 2\sqrt{2} = 16\sqrt{2} \]

#2/10 Solve (set): \( e^{2x} + 2e^x - 8 = 0 \) Let \( A = e^x \)

\[ \therefore A^2 + 2A - 8 = 0 \therefore (A+4)(A-2) = 0 \]

\[ \therefore A = -4 \text{ or } A = 2 \] (Exclude -4) \[ ; e^x = 2 \]

\[ ; x = \ln(2) \] \[ \{ \ln(2) \} \]
\#3/10  Solve for \( x \). Exact.
\[
\ln(x) + \ln(x-1) = \ln(6)
\]
\[\text{Solve: } \ln(x(x-1)) = \ln(6) \Rightarrow x(x-1) = 6
\]
\[\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0
\]
\[\Rightarrow x = 3 \text{ or } x = -2 \quad \text{Exclude } x = -2
\]
\[\Rightarrow \text{Sol set is } \{3\}
\]
\#4/10  \( M = \log \left( \frac{I}{s} \right) \); \( M_{SF} = \log \left( \frac{I_{SF}}{s} \right) = 8.3 \)
and \( M_I = \log \left( \frac{I_I}{s} \right) \) and \( I_T = 5I_{SF} \)

So \( M_T = \log \left( \frac{I_T}{s} \right) = \log \left( \frac{5I_{SF}}{s} \right) \)

\[= \log \left( 5 \cdot \frac{I_{SF}}{s} \right) = \log (5) + \log \left( \frac{I_{SF}}{s} \right)
\]

\[= \log (5) + 8.3 \approx 8.998970004 \approx 9.0
\]

The magnitude of the Tallahassee earthquake was about 9.0 on the Richter scale.
#5/10\[ \begin{aligned} x + 3z &= 3 \\ 2x + y - 2z &= 5 \\ -y + 8z &= 8 \end{aligned} \rightarrow \begin{aligned} x + 3z &= 0 \\ y - 8z &= 0 \\ 0 &= 1 \end{aligned} \]

Since the reduced form includes the equation 0=1, which is impossible, the system is inconsistent.

#6/10\[ \begin{aligned} x + z &= 2 \\ 2x + y + z &= 5 \\ 3y - 3z &= 3 \end{aligned} \rightarrow \begin{aligned} x + z &= 2 \\ y - z &= 1 \\ 0 &= 0 \end{aligned} \]

\[ \therefore \begin{aligned} x &= 2 - z \\ y &= 1 + z \\ z &= z \end{aligned} \]

\[ \begin{aligned} x &= 2 - t \\ y &= 1 + t \\ z &= t \end{aligned} \]

The system is dependent.

\[ \therefore \text{Sol set } \{(x,y,z) | x=2-t, y=1+t, z=t, t \in \mathbb{R}\} \]

#7/10 Solve, No calc.

\[ \begin{aligned} x + y &= 20 \\ xy &= 91 \end{aligned} \rightarrow \begin{aligned} y &= 20 - x \\ (20-x)^2 &= 91 \end{aligned} \]

\[ \therefore \begin{aligned} -x^2 + 20x - 91 &= 0 \\ x^2 - 20x + 91 &= 0 \end{aligned} \]

(Complete the square) \[ \begin{aligned} x^2 - 20x &= -91 \\ x^2 - 20x + 100 &= -91 + 100 = 9 \end{aligned} \]

\[ \therefore (x-10)^2 = 9 \]

\[ \therefore x - 10 = \pm 3 \]

\[ \therefore x = 10 \pm 3 \]

\[ \therefore x = 13 \text{ and } y = 7 \text{ or } x = 7 \text{ and } y = 13 \]

\[ \therefore \text{Sol set. } \{(13,7), (7,13)\} \]
Let \( x \) = \# of gallons of reg. gas.

\[ y = "" "" "" hi-test."" \]

1. \( x + y = 5 \)
2. \( 2.00x + 3.00y = 13.50 \)

\[ y = 5 - x \]

\[ 2x + 3(5-x) = 13.5 \]

\[ 2x + 15 - 3x = 13.5 \]

\[ 15 - 13.5 = x \quad \text{so} \quad x = 1.5 \]

\[ y = 5 - x = 5 - 1.5 = 3.5 \]

\[ y = 3.5 \]

I pumped 1.5 gal. of regular gas and 3.5 gal. of hi-test gas.

Check:
\[ 2(1.5) + 3(3.5) = 3 + 10.5 = 13.5 \checkmark \]

\[ \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 5 \\ 9 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 11 \end{bmatrix} \]

\[ \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 5 \\ 9 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 11 \end{bmatrix} \]

\#10/10| Solve, Exact \( \frac{30}{2 + 3e^{-x}} = 10 \)

\[ 30 = 10(2 + 3e^{-x}) \quad \text{so} \quad 3 = 2 + 3e^{-x} \]

\[ 1 = 3e^{-x} \quad \frac{1}{3} = e^{-x} \quad \frac{1}{x} = \frac{1}{e^x} \quad e^x = 3 \]

\[ x = \ln(3) \quad \{ \ln(3) \} \]

**BONUS**

\[ n(t) = n_0e^{rt} \quad \text{at} \quad n(t) = 85e^{0.18t} \]

\[ n(3) = 85e^{0.54} \approx 145 \quad 860 \quad 5833 \approx 146 \text{ frogs.} \]

\[ 600 = 85e^{0.18t} \quad \frac{600}{85} = e^{0.18t} \]

\[ t = \frac{100}{0.18} \cdot \ln \left( \frac{120}{17} \right) = \frac{50}{0.18} \cdot \ln \left( \frac{120}{17} \right) \approx 10.85710222 \text{ yrs.} \]