\[
\frac{2x^5 - 7x^3 + 2x^2 + 6x + 1}{x^2 + 1}
\]

Solution:
\[
\frac{2x^3 - 9x + 2}{x^2 + 1}
\]
\[
= \frac{2x^5 + 0x^4 - 7x^3 + 2x^2 + 6x + 1}{2x^5 + 2x^3}
\]
\[
\overset{2x^3}{\underset{2x^5}{\theta}}
\]
\[
\theta 9x^3 \quad \theta 9x
\]
\[
\theta 9x^3 \quad \theta 2
\]
\[
\theta 2x^2
\]
\[
\theta (15x - 1)
\]
\[
\therefore \frac{2x^5 - 7x^3 + 2x^2 + 6x + 1}{x^2 + 1} = 2x^3 - 9x + 2 + \frac{15x - 1}{x^2 + 1}
\]

2. \[\text{S.D. } P(x) = D(xQ(x)) + R(x)\]
\[
x^7 + 2x^6 + x^4 - 4x^2 + 7x - 18
\]
\[
\overset{2}{x + 2}
\]

Solution:
\[
\begin{array}{cccccc}
-2 & 1 & 2 & 0 & 1 & 0 - 4 & 7 - 18 \\
-2 & 0 & 0 & 2 & 4 & 0 & 14 \\
1 & 0 & 0 & 1 & -2 & 0 & 7 & -32
\end{array}
\]

\[
\therefore x^7 + 2x^6 + x^4 - 4x^2 + 7x - 18
\]
\[
= (x + 2)(x^6 + x^5 - 2x^2 + 7) - 32
\]
Use SD & RemThm. Find $P(c)$ if $P(x) = 4x^3 + 5x - 8$, $c = -\frac{1}{5}$.

So! $\begin{vmatrix} 1 & -\frac{1}{5} & 0 & \frac{125}{5} & -8 \\ 0 & \frac{4}{5} & \frac{129}{25} & -\frac{129}{25} \\ \frac{1220}{125} & -\frac{4}{5} & \frac{4}{25} & -\frac{129}{125} & -\frac{8}{1} = -\frac{1000}{125} \end{vmatrix}$

\[ P(\frac{1}{5}) = \frac{1220}{125} \]

\[ \frac{1220}{125} - \frac{4}{5} \cdot \frac{4}{25} - \frac{129}{125} \]

\[ 4 - \frac{4}{5} = \frac{1229}{125} - \frac{1120}{125} \]

\[ \frac{129}{125} - \frac{8}{1} = -\frac{1000}{125} \]

\[ \frac{1}{5} = \frac{7-6i}{5} = \frac{7}{5} - \frac{6}{5}i \]

\[ \frac{7}{5} - \frac{6}{5}i \]

\[ \frac{2}{3-2i} - \frac{3}{3+2i} = \frac{2(3+2i)}{(3-2i)(3+2i)} \]

\[ \frac{6+4i-9+6i}{9+4} = \frac{-3+10i}{13} = \frac{-3}{13} + \frac{10}{13}i \]

All sols. (set) $y = 4 - \frac{7}{9}$

10 $y^2 = 4y - 7$

$c = 1, b = -4, c = 7$

\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ y = \frac{4 \pm \sqrt{16-4(7)}}{2} = \frac{4 \pm 2\sqrt{-3}}{2} = 2 \pm \sqrt{3}i \text{ or } 2 \pm i\sqrt{3} \]

Sol. set \{2 + i\sqrt{3}, 2 - i\sqrt{3}\}
\( \text{\#6} \quad \text{(5)} \quad \text{deg} P=3, \text{ zeros are } 2-i, 4. \)

Find \( P(x) \).

Solution \#1: "Other Zero" is \( 2 + i \).

\[ P(x) = (x-4)(x-(2-i))(x-(2+i)) \]

So \[ x = 2 \pm i \Rightarrow x-2 = \pm i \]
\[ \Rightarrow (x-2)^2 = (-i)^2 = -1 \Rightarrow x^2 - 4x + 5 = 0 \Rightarrow \text{ \( x^2 - 4x + 5 \) is a zero} \]

\[ \therefore P(x) = (x-4)(x^2 - 4x + 5) \]

\[ = x^3 - 4x^2 + 5x - 4x^2 + 16x - 20 \]

\[ \therefore P(x) = x^3 - 8x^2 + 21x - 20 \]

\( \text{\#7: Factor. Find all zeros. State "multi."} \)

\[ P(x) = x^3 + 9x^2 + 15x - 25 \]

Solution \#1: Pass Rat. Zeros \( \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{25}{3} \)

Solution \#2: I think \( x=1 \) "works." Let's see.

\[ \begin{array}{c|cccc} \hline x & 1 & 9 & 15 & -25 \\ \hline 1 & 0 & 25 & 0 & \text{Yay!} \\ 1 & 0 & 25 & 0 & \hline \end{array} \]

\[ \therefore P(x) = \frac{(x-1)(x^2 + 10x + 25)}{(x-1)(x+5)} \text{ or } P(x) = (x-1)(x+5)^2 \]

Factored completely.

\[ \text{\#4: The zeros are } 1 \text{ and } -5 \text{ (multi. 2).} \]

\[ \text{\#8: Bonus} \quad S_n = \frac{a}{1-r}, \text{ so } \sum_{k=1}^{n} \left( \frac{1}{3} \right)^{k-1} = \frac{4}{1-\frac{1}{3}} \]

\[ = \frac{4}{\frac{2}{3}} = \frac{4 \cdot \frac{3}{2}}{1} = \boxed{6} \]