The "vertical parabola" is the set of all points \( P(x,y) \) which are **equidistant** from the line \( y = -p \) (the directrix) and the point \( F(0,p) \) (the focus). [In this figure we have taken \( p > 0 \). The case for \( p < 0 \) is similar, except that the parabola would then open downward.]

Let's look at the algebra: \( d_1 = d_2 \) (by definition). So \( d(PQ) = d(PF) \) and \( P(x,y), Q(x,-p), F(0,p) \)

\[
\sqrt{(x-x)^2 + (y-(-p))^2} = \sqrt{(x-0)^2 + (y-p)^2} \\
\text{(Simplify)} \\
\sqrt{(y+p)^2} = \sqrt{x^2 + (y-p)^2} \\
\text{(Square both sides)} \\
(y+p)^2 = x^2 + (y-p)^2 \\
\text{(Expand binomials)} \\
y^2 + 2py + p^2 = x^2 + y^2 - 2py + p^2 \\
2py = x^2 - 2py \\
\text{(Isolate } x^2) \\
4py = x^2 \\
\text{and re-write: } x^2 = 4py \\
\] 
This is **NSF** (New Standard Form) for the parabola.