HELP! in MAC 2140

• #1 [§ 10.2: p.760: # 25] Use Graphing Device to graph

\[ \frac{x^2}{25} + \frac{y^2}{20} = 1. \]

**Solution**

1. To enter this into the graphing calculator — solve for \( y \):

\[ \frac{x^2}{25} + \frac{y^2}{20} = 1 \implies \frac{y^2}{20} = 1 - \frac{x^2}{25} \]

\[ \implies y^2 = 20\left(1 - \frac{x^2}{25}\right) \implies y = \pm \sqrt{20\left(1 - \frac{x^2}{25}\right)} \]

2. So

\[ y_1 = \sqrt{20\left(1 - \frac{x^2}{25}\right)} \quad \text{and} \quad y_2 = -\sqrt{20\left(1 - \frac{x^2}{25}\right)} \]

is one way to enter the ellipse into your calculator.

• #2 [§ 10.2: p.760: # 37] Find Eq. for ellipse w/ length of major axis is 10; foci on x-axis; ellipse passes through \((\sqrt{5}, 2)\).

**Solution**

1. Length of major axis is \(2a\), \(2a = 10\) \(\Rightarrow a = 5\)

2. Foci on x-axis \(\Rightarrow\) ellipse is HORIZONTAL \(\sqrt{a^2} + \sqrt{\frac{y^2}{b^2}} = 1\)

\[ \frac{x^2}{25} + \frac{y^2}{b^2} = 1 \]

3. Ellipse passes through \((\sqrt{5}, 2)\) \(\Rightarrow\) \[ \frac{5}{25} + \frac{4}{b^2} = 1 \]

\[ \text{Solve this for } b. \]

4. \[ \frac{1}{5} + \frac{4}{b^2} = 1 \implies \frac{4}{b^2} = 1 - \frac{1}{5} = \frac{4}{5} \implies \frac{b^2}{4} = \frac{5}{4} \implies b^2 = 5 \]

If 2 fractions are equal and their numerators are equal, then their denominators are equal.

\[ \frac{x^2}{25} + \frac{y^2}{5} = 1 \]
# 3 [§ 10.2: p. 760: # 39] Find eq. for ellipse, eccentricity is 0.8. Foci (±1.5, 0)

**Sol.**

1. Foci (±1.5, 0) gives me two "things."
   1. The ellipse is horizontal \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and
   2. \( c = 1.5 \)
   3. \( e = \frac{c}{a} \) (Fact of Life) \( e = 0.8 \) (Fact-of-the-Problem)

\[ 0.8 = \frac{1.5}{a} \quad \Rightarrow \quad a = \frac{1.5}{0.8} = \frac{15}{8} \]

\[ a^2 = \frac{225}{64} - \frac{2.25}{x} \]

\[ b^2 = a^2 - c^2 = \frac{225}{64} - \frac{144}{64} = \frac{81}{64} \]

\( a^2 = \frac{225}{64}, \quad b^2 = \frac{81}{64} \)

4. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) \( \frac{x^2}{\frac{225}{64}} + \frac{y^2}{\frac{81}{64}} = 1 \)

# 4 [§ 10.2: p. 760: # 44] The ANCILLARY CIRCLE of an ellipse is the circle with radius \( \frac{1}{2} \) the length of the minor axis & center same as that of the ellipse. The ancillary circle is thus the largest circle which can fit within the ellipse. **(2) If** \( x^2 + 4y^2 = 16 \), **find the eq. of the ancillary circle.**

**Soln.** \( \frac{x^2}{16} + \frac{4y^2}{16} = 1 \) \( \frac{x^2}{16} + \frac{y^2}{4} = 1 \) This is a horiz. ellipse.

We see that \( b^2 = 4 \), so \( b = 2 \) so the radius of the ancill. circle is \( r = 2 \) (same as \( b \)). The center of the ellips is \( (0,0) \). Thus so is the center of the ancill.circ. Thus the eq is \( x^2 + y^2 = 4 \)

(6) Show that if \( (5,t) \) is a point on the ancill. circle, then \( (25,t) \) is on the ellipse.
Proof: Suppose \((s,t)\) is on the circle \(x^2 + y^2 = 4\). Then it follows that \(s^2 + t^2 = 4\).

From this it follows that \(t^2 = 4 - s^2\).

Next consider the ellipse \(x^2 + 4y^2 = 16\). If I substitute \(t\) for the \(y\)-coordinate in the ellipse, I get:

\[
x^2 + 4t^2 = 16 - \text{Now subs } t^2 = 4 - s^2
\]

\[
x^2 + 4(4 - s^2) = 16
\]

\[
x^2 + 16 - 4s^2 = 16
\]

\[
x^2 - 4s^2 = 0
\]

\[
x^2 = 4s^2
\]

\[
x = \pm 2s
\]

\[
\therefore \quad (2s, t) \text{ is a point on the ellipse.}
\]

\[\#5 \quad \#10.2; p. 761; \#51\] Sunburst Window. A sunburst window is constructed in the shape of the top half of an ellipse. The window is 20 in tall at its highest point and 80 in wide at the bottom. Find the height of the window 25 in from the center of the base.

![Sunburst Window Diagram]

Solution 1: \(a = 40, \ b = 20\) so \(\frac{x^2}{40^2} + \frac{y^2}{20^2} = 1\). If \(x = 25\) we wish to find \(y\), which will be the height of the window.

\[
2 \left(\frac{25^2}{40^2} + \frac{y^2}{20^2}\right) = 1 \quad \therefore \quad \frac{y^2}{20^2} = 1 - \frac{25^2}{40^2} = \frac{40^2 - 25^2}{40^2}
\]

\[
y^2 = \frac{40^2 - 25^2}{40^2}
\]

\[
y = \pm \sqrt{\frac{25}{4}} (39) = \pm \frac{5}{2} \sqrt{39}.
\]

\[\therefore \quad \text{The height is } \frac{5}{2} \sqrt{39} \text{ in.}\]
**Problem:** The latus rectum for an ellipse is a line segment perpendicular to the major axis at a focus with endpoints on the ellipse. Show that the length of a latus rectum is \( \frac{2b^2}{a} \).

**Solution:**

1. We wish to find the length of \( AB \), \( d(AB) \).
2. This is a horizontal ellipse: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (4). \( F_2(c,0) \) is a focus.
3. So the \( x \)-coordinate of both \( A \) and \( B \) is \( c \). To find the \( y \)-coordinates of \( A \) and \( B \), we substitute \( c \) into eq. (4) and solve for \( y \).

\[
\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{y^2}{b^2} = 1 - \frac{c^2}{a^2} = \frac{a^2 - c^2}{a^2} = \frac{b^2}{a^2} \quad \text{(since } b^2 = a^2 - c^2)\]

So \( \frac{y^2}{b^2} = \frac{b^2}{a^2} \). Thus \( y^2 = \frac{b^2}{a^2} \), so \( y = \pm \frac{b}{a} \).

4. Thus \( d(AB) = \sqrt{(c - c)^2 + (\frac{b^2}{a} - (-\frac{b^2}{a}))^2} = \sqrt{0 + (\frac{b^2}{a} + \frac{b^2}{a})^2} \)

\[
= \sqrt{(\frac{2b^2}{a})^2} = \frac{2b^2}{a} \quad \text{(since } b > 0, a > 0)\]

\[\therefore d(A,B) = \frac{2b^2}{a} \]