A fish farmer has 5000 catfish in his pond. The number of catfish increases by 8% per month, and the farmer harvests 300 catfish per month.

(a) Show that the catfish population, \( P_n \), after \( n \) months is given recursively by

\[ P_n = 1.08 P_{n-1} - 300, \quad P_0 = 5000. \]

(b) How many fish are in the pond after 12 months?

Solution

1. Every month the population increases by \( 8\% = 0.08 \). Thus if \( P_{n-1} \) is the population at the end of the previous month, then the population at the end of this month \( P_n \) would be

\[ P_n = P_{n-1} + 0.08 P_{n-1} \]

So \( P_n = 1.08 P_{n-1} \). If no catfish were being harvested, (subtracted) from the pond.

2. However, 300 catfish are being harvested. Thus the recursion formula is

\[ P_n = 1.08 P_{n-1} - 300. \]

We have also the "start-up" data \( P_0 = 5000 \). (Notice that we start \( P_0 \) with subscript zero in this problem.

3. Thus \( P_n = 1.08 P_{n-1} - 300 \). \( P_0 = 5000. \)

(b) The only way we have to work this part at this stage of our mathematical development is to iterate. But let's do it using our calculators—without rounding off until the final step.

Type 5000 and [ENTER]. Type 1.08 [ANS] - 300 and [ENTER].

Then just keep pushing [ENTER].

\[ P_5 = 5324.64 \quad P_6 = 5733.92904 \quad P_7 = 6164.96713 \]

The result is:

After 12 mo., there are approximately 6897 fish in the pond.