#1) 5.9.2: p. 649: #45: Value of Coins.

A man has 14 coins in his pocket, all of which are dimes and quarters. If the total value of his change is $2.75, how many dimes and how many quarters does he have?

Solution: The idea here is to come up with a technique which will generalize for solving more complicated problems. You could figure out this problem on a "trial and error" basis—but that's not what we're looking for AND that would take too long for a test!

Here's the idea: We've got TWO concepts going for us here: (a) The \textbf{actual number} of each kind of coin and (b) The \textbf{value} of each kind of coin.

So we have two unknowns (the \textbf{number of dimes} and the \textbf{number of quarters}) and we have two equations (the "coin" equation and the "value" equation).

Here's how it works:

\begin{enumerate}
  \item Let \(D\) = the number of dimes in his pocket. \text{AND} \quad \(Q\) = the number of quarters.
  \item The coin eq.: \(D + Q = 14\) (he has 14 coins total)
  \item The value eq.: \(0.10D + 0.25Q = 2.75\) (the value of the dimes plus the value of the quarters = $2.75.
\end{enumerate}

\begin{enumerate}
  \item Do the Math:
  \[
  \begin{align*}
  D + Q &= 14 \quad \text{(1)} \\
  0.10D + 0.25Q &= 2.75 \quad \text{(2)}
  \end{align*}
  \]
  \[
  \begin{align*}
  10D + 10Q &= 140 \quad \text{(3)} \\
  \frac{10D}{10} + \frac{25Q}{10} &= \frac{275}{10} \\
  D + \frac{2.5Q}{1} &= 27.5 \\
  \frac{2.5Q}{2.5} &= \frac{27.5}{2.5} \\
  Q &= 11 \frac{1}{2}
  \end{align*}
  \]
  \[
  \begin{align*}
  -10Q &= -115 \quad \text{(4)} \\
  10D - 10Q &= -1.4 \quad \text{(5)}
  \end{align*}
  \]
  \[
  \begin{align*}
  D - Q &= -1.4 \\
  \frac{D}{1} + \frac{-Q}{1} &= \frac{-1.4}{1} \\
  D - Q &= -1.4 \\
  \frac{D}{1} - \frac{Q}{1} &= \frac{-1.4}{1} \\
  \frac{D}{1} - \frac{Q}{1} &= \frac{1.4}{1} \\
  D - Q &= 1.4 \\
  \end{align*}
  \]

So \(Q = 11 \frac{1}{2} = 11.5 = 9\) and, since \(D + Q = 14\), \(D\) must be 5.

The man has 9 quarters and 5 dimes for a sum of $2.25 + $0.50 = $2.75.