Prove the statement using the \( \varepsilon, \delta \) definition of limit and illustrate with a diagram like Figure 9.

\[
\lim_{x \to 4} (7-3x) = -5
\]

**Solution:**

1. Let \( \varepsilon > 0 \) be assigned. I must find \( \delta > 0 \) such that

\[
0 < |x-4| < \delta \implies |f(x) - L| < \varepsilon.
\]

In this case

\[
0 < |x-4| < \delta \implies |(7-3x)+5| < \varepsilon.
\]

2. Work: Working with the LHS of the \( \varepsilon \)-inequality

\[
|(7-3x)+5| = |12-3x| = 3|4-x| = 3|x-4| < 3\delta
\]

Set \( \varepsilon > \frac{\delta}{3} \)

3. Let \( \delta = \frac{\varepsilon}{3} \)

4. Proof: \( \varepsilon > 0 \) has been given. Let \( \delta = \frac{\varepsilon}{3} \) be the corresponding \( \delta \). Then

if \( 0 < |x-4| < \delta = \frac{\varepsilon}{3} \), then it follows that

\[
|(7-3x)+5| = |12-3x| = 3|4-x| = 3|x-4| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon.
\]

\( \therefore |(7-3x)+5| < \varepsilon \).

Thus

\[
\lim_{x \to 4} (7-3x) = -5.
\]