Find the length of the curve \( y = \sqrt{1-x^2} \) over \(-1 \leq x \leq 1\).

**Solution**: The graph is the upper semicircle of radius 1, so I already know the answer is \( \pi \) units, but let's go through the process.

\[ f(x) = (1-x^2)^{1/2}, \quad f'(x) = \frac{1}{2} (1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{(1-x^2)^{1/2}} \]

and \( \left[ f'(x) \right]^2 = \frac{x^2}{1-x^2} \)

So \[ \sqrt{1 + \left[ f'(x) \right]^2} = \sqrt{1 + \frac{x^2}{1-x^2}} = \sqrt{\frac{(1-x^2) + x^2}{1-x^2}} = \sqrt{\frac{1}{1-x^2}} = \frac{1}{\sqrt{1-x^2}} \]

The formula for arc-length (length of the curve) is

\[ L = \int_{x=a}^{x=b} \sqrt{1 + [f'(x)]^2} \, dx \]

Thus \[ L = \int_{x=-1}^{x=1} \frac{1}{\sqrt{1-x^2}} \, dx \]

Let \( x = \cos(t) \)
\[ dx = -\sin(t) \, dt \]
\[ x^2 = \cos^2(t) \]
\[ 1-x^2 = 1-\cos^2(t) = \sin^2(t) \]
\[ x = -1, \quad t = \pi \]
\[ x = 1, \quad t = 0 \]

Thus the length of the curve is \( \pi \) units.

\[ \int_{t=0}^{t=\pi} - \, dt = \int_{t=0}^{t=\pi} \, dt = \pi \]