6.4: p. 463: #6 A thin rod of density \( \delta(x) = 4 \) lies on the interval \([1, 3]\) of the \(x\)-axis. Find \( M_0, M, \) and C.M.

**Solution**

1. Sketch:

2. \[
M = \int_{1}^{3} \delta(x) \, dx = \int_{1}^{3} 4 \, dx = 4 \left[ x \right]_{1}^{3} = 8 \text{ units of mass}.
\]

3. \[
M_0 = \int_{1}^{3} x \delta(x) \, dx = \int_{1}^{3} 4x \, dx = \left[ 2x^2 \right]_{1}^{3} = 2(9-1) = 16 \text{ mass \cdot length}.
\]

4. \[
\bar{x} = \frac{M_0}{M} = \frac{16 \text{ mass \cdot length}}{8 \text{ mass}} = 2 \text{ units (length)}.
\]

5. \( M_0 = 16 \text{ mass \cdot length}, \) \( M = 8 \text{ mass}, \) \( \bar{x} = 2 \text{ units (length)} \)