#1 [6.1: p. 435: #5] Find the volume of the solid which lies between planes \( \perp \) to the \( x \)-axis @ \( x = -1 \) \& \( x = 1 \). The cross-sections \( \perp \) to the \( x \)-axis are squares whose bases run from the semicircle \( y = -\sqrt{1-x^2} \) to the semicircle \( y = \sqrt{1-x^2} \).

**Solution:**

1. **Sketch.**
   - **A**: \( x^2 + y^2 = 1 \)
   - **B**: \( y = -\sqrt{1-x^2} \)
   - **C**: \( y = \sqrt{1-x^2} \)
   - **D**: \( x \)
   - **E**: \( y \)
   - **F**: \( z \)

2. **\( \Delta \) slice is \( \Delta x \)**

3. **Area of Cross-section:** \( A(x) = s^2 \)
   - but we must express \( s \) in terms of \( x \).
   - Here's the reasoning: \( C \) is a "generic point" on the \( x \)-axis. Its coordinates are \((x, 0)\). More importantly, the coordinates of \( A \) and \( B \) are \( A(x, -\sqrt{1-x^2}) \) and \( B(x, \sqrt{1-x^2}) \) and the distance between \( A \) and \( B \) is pretty obviously \( 2\sqrt{1-x^2} \). But this is \( s \).

   \[ A(x) = \left[ 2\sqrt{1-x^2} \right]^2 = 4(1-x^2) \text{ (by symmetry)} \]

4. **\( V = \int_{x=a}^{x=b} A(x) \, dx = \int_{x=-1}^{x=1} 4 \left( 1-x^2 \right) \, dx = 2 \int_{y=0}^{y=1} 4 \left( 1-y^2 \right) \, dy \)**
   - \( = 8 \left[ x - \frac{x^3}{3} \right]_{x=-1}^{x=1} = 8 \left[ 1 - \frac{1}{3} \right] = 8 \cdot \frac{2}{3} = \frac{16}{3} \text{ units}^3 \)

   **Thank You, Sir Isaac!**

5. **The volume is \( \frac{16}{3} \text{ units}^3 \)**
#2  [p. 436: #23] Find vol. of solid of rev. gen. by rotating region \( R \) abt. \( x \)-axis.

\[ R: \quad y = \sqrt{\cos(x)}, \quad x \in \left[ 0, \frac{\pi}{2} \right], \quad y = 0, \quad x = 0. \]

**Soln.**

1. Sketch.

   ![Sketch of the region](image)

   **Note:** The \( x \)-axis is at the center of the circle & it is perpendicular to this page.

2. \( \Delta \) slice: \( \Delta x \)

3. \( A(x) = \pi [r(x)]^2 \)
   \[ A(x) = \pi \left( \sqrt{\cos(x)} \right)^2 \]
   \[ A(x) = \pi \cos(x) \]

4. \[ V = \int_{x=a}^{x=b} A(x) \, dx = \int_{x=0}^{x=\frac{\pi}{2}} \pi \cos(x) \, dx \]
   \[ = \pi \int_{x=0}^{x=\frac{\pi}{2}} \cos(x) \, dx = \pi \left\{ \sin(x) \right\}_{x=0}^{x=\frac{\pi}{2}} = \pi \left\{ 1 - 0 \right\} = \pi \text{ units}^3 \]

5. The volume is \( \pi \) units\(^3\).