
Let D be the region bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 2y \). Write triple iterated integrals in the order \( dz \, dx \, dy \) and \( dz \, dy \, dx \) that give the volume of D. Do not evaluate either integral.

**Solution**

1. Let's get some idea of the graph.

   a) If \( x = 0 \) (looking at the projection of D onto the \( yz \)-plane):

   \[
   \begin{align*}
   z &= x^2 + y^2 \\
   z &= 2y
   \end{align*}
   \]

   \[
   \Rightarrow \quad x^2 + y^2 = 2y \quad \Rightarrow \quad x^2 + y^2 - 2y = 0
   \]

   \[
   \Rightarrow \quad x^2 + (y^2 - 2y + 1) = 1 \quad \Rightarrow \quad (x^2 + (y-1)^2 = 1)
   \]

   So the projection is a circle.

   b) But now, more importantly, since we first must deal with \( \int dz \, dx \, dy \) let's look at the projection of D onto the \( xy \)-plane:

   Analytically, we eliminate \( z \) from our bounding equations:

   \[
   \begin{align*}
   x^2 + y^2 &= 2y \\
   z &= 2y
   \end{align*}
   \]

   \[
   \Rightarrow \quad x^2 + (y^2 - 2y + 1) = 1 \quad \Rightarrow \quad (x^2 + (y-1)^2 = 1)
   \]

   \[
   \text{For} \ (y \text{ fixed}) \ 	ext{z varies from} \ z = x^2 + y^2 \ 	ext{to} \ \ z = 2y \ .
   \]

   \[
   \text{For} \ \frac{y}{2} \ \text{fixed} \ x \ 	ext{varies from} \ x = -\sqrt{2y-y^2} \ 	ext{to} \ x = \sqrt{2y-y^2}
   \]

   \[
   \text{y varies from} \ y = 0 \ 	ext{to} \ y = 2 .
   \]
\[ V = \int_{y=0}^{y=2} \int_{x=-\sqrt{2y-y^2}}^{x=\sqrt{2y-y^2}} \int_{z=x+y^2}^{z=2y} dz\,dx\,dy. \]

2. Consider the order \( dz\,dy\,dx \).

\[ \text{(a)} \quad \text{For } (x,y) \text{ fixed}, \quad z \text{ varies from } z=x^2+y^2 \text{ to } z=2y. \]
\[ \text{For } x \text{ fixed } \quad y \text{ varies from } y=1-\sqrt{1-x^2} \text{ to } y=1+\sqrt{1-x^2} \]
\[ x \text{ varies from } x=-1 \text{ to } x=1 \]

\[ V = \int_{x=-1}^{x=1} \int_{y=1-\sqrt{1-x^2}}^{y=1+\sqrt{1-x^2}} \int_{z=x^2+y^2}^{z=2y} dz\,dy\,dx. \]