CALC III

Take-off from §15.4; p.1091: #22.

Given: \( \int_0^1 \int_{-1}^1 \int_0^{y^2} dz \, dy \, dx \)

Required:

\[ \text{Sketch the domain } D. \]

\[ \text{Re-write the triple iterated integral in the order} \]

\[ i) \, dy \, dz \, dx, \quad ii) \, dy \, dx \, dz, \quad iii) \, dz \, dy \, dx \]

\[ \square \, dx \, dz \, dy \]

Soln:  

\[ \text{The initial order is } dz \, dy \, dx, \text{ so} \]

As \( y \) and \( z \) are fixed \( (xy) \) is fixed, \( z \) varies from \( z = 0 \) to \( z = y^2 \)

As \( z \) is fixed, \( y \) varies from \(-1\) to \( 0 \), and \( x \) varies from \( 0 \) to \( 1 \).
Putting the graphical facts together:

As $x$ and $z$ are fixed, $y$ varies from $y = -1$ to $y = \sqrt{z}$.

As $x$ is fixed, $z$ varies from $z = 0$ to $z = 1$.

$x$ varies from $x = 0$ to $x = 1$.

\[ I = \int_{x=0}^{x=1} \int_{z=0}^{z=1} \int_{y=-1}^{y=\sqrt{z}} dy \, dz \, dx \]

$dy \, dx \, dz$:

For $z$ and $x$ fixed, $y$ varies from $y = -1$ to $y = -\sqrt{z}$.

For $z$ fixed, $x$ varies from $x = 0$ to $x = 1$.

$z$ varies from $0$ to $1$.

\[ I = \int_{z=0}^{z=1} \int_{x=0}^{x=1} \int_{y=-1}^{y=-\sqrt{z}} dy \, dx \, dz \]
I'll leave the rest of these to you. By the way... it's easy (?) to check to see if you are right because all of these integrals should evaluate to the same value:

\[
\int_{x=0}^{x=1} \int_{y=-1}^{y=0} z \cdot y^2 \, dz \, dy \, dx = \int_{x=0}^{x=1} \int_{y=-1}^{y=0} y^2 \, dy \, dx
\]

\[
= \frac{1}{3} \int_{x=c}^{x=1} \left( y^3 \right)_{y=-1}^{y=0} \, dx = \frac{1}{3} \int_{x=c}^{x=1} 1 \, dx = \frac{1}{3}
\]