§12.2: p. 845: # 41: LINEAR COMBINATION. Let \( \vec{u} = 2\vec{e} + \vec{j} \) and \( \vec{v} = \vec{e} + \vec{j} \) and \( \vec{w} = \vec{e} - \vec{j} \). Find scalars \( a \) and \( b \) such that 

\[ \vec{u} = a\vec{v} + b\vec{w} \]

So,

1. I'll use "\( \langle \cdot, \cdot \rangle \)" notation in my work area to save writing.

2. Assume we have found \( a \) and \( b \), but we don't know yet what they are, i.e., Assume \( a \) and \( b \) exist.

Then

\[ \vec{u} = a\vec{v} + b\vec{w} \]

or

\[ \langle 2, 1 \rangle = a\langle 1, 1 \rangle + b\langle 1, -1 \rangle \]

\[ = \langle a, a \rangle + \langle b, -b \rangle \]

\[ = \langle a+b, a-b \rangle \]

3. Now two vectors are equal iff they are component-wise equal.

\[ \therefore \begin{align*}
    a+b &= 2 \\
    a-b &= 1
\end{align*} \]

\( a+b = 2 \) is a system of 2 eq's in 2 unknowns.

\( a-b = 1 \) Solution is easy.

4. \[ \begin{align*}
    a+b &= 2 \\
    a-b &= 1
\end{align*} \]

\[ 2a = 3 \]

\[ a = \frac{3}{2} \] and (by observation) \( b = \frac{1}{2} \)

5. \[ \vec{u} = \frac{3}{2} \vec{v} + \frac{1}{2} \vec{w} \]