A golf ball leaves the ground at a $30^\circ$ angle and at a speed of 90 fps (ft/sec). Assume $g = 32$ ft/sec$^2$.

(a) State the IPME (Ideal Projectile Motion Equation) which is in vector form, and

(b) Use this equation to determine if the golf ball will clear the top of a 28-ft pine tree which is 135 ft down the fairway. Explain with a final sentence.

**Solution:**

(a) $\vec{r}(t) = (v_0 \cos \alpha) t \hat{i} + ((v_0 \sin \alpha) t - \frac{1}{2} gt^2) \hat{j}$

(b) If $v_0 = 90$ ft/sec, $\alpha = 30^\circ$ and $x = 135$

\[ t = \frac{x}{v_0 \cos \alpha} = \frac{135}{90 \cos 30^\circ} = \frac{3}{2} \cdot \frac{2}{\sqrt{3}} = \sqrt{3} \text{ sec} \]

\[ \text{Ht of golfball at } x = 135 \text{ ft is } y \]

\[ y = (v_0 \sin \alpha) t - \frac{1}{2} gt^2 = (90 \sin 30^\circ) \sqrt{3} - 16.3 \]

\[ = 45 \sqrt{3} - 48 \approx 29.942 \text{ ft} > 28 \text{ ft.} \]

At 135 ft downrange, the height of the golf ball will be over 29 ft; therefore, it will clear the 28-ft pine tree.
23.

#2
(a) Use the formula \( s(t) = \int_{t_0}^{t} \| \mathbf{r}'(t) \| \, dt \)

to find the "arc length parameter" \( s(t) \)
if \( \mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, 12t \rangle \) (helix)

(b) Use the result of (a) to find the length of
the helix if \( 0 \leq t \leq \pi \). (Units are km).

**Solution**
(a) \( \mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, 12t \rangle \)
\( \mathbf{r}'(t) = \langle -5 \sin t, 5 \cos t, 12 \rangle \quad \text{and} \)
\( \| \mathbf{r}'(t) \| = \sqrt{(-5 \sin t)^2 + (5 \cos t)^2 + 144} = (25 \sin^2 t + 25 \cos^2 t + 144)^{1/2} = (169)^{1/2} = 13 \)

(b) \( s(t) = \int_{t_0}^{t} \| \mathbf{r}'(t) \| \, dt = 13t \quad \text{(units km)} \)

\[ L = s(\pi) = 13\pi \text{ km} \]

#3
Find the curvature of \( f(x) = x^2 \) when \( x = 3 \).
Give exact answer and approx. answer to 10 decimal places.

You may wish to use one of these formulas:

(a) \( \kappa = \frac{1}{\| \mathbf{r}'(t) \|} \left\| \frac{d^2 \mathbf{r}(t)}{dt^2} \right\| \)

(b) \( \kappa = \frac{1}{\left( 1 + (f'(x))^2 \right)^{3/2}} \frac{f''(x)}{\left( x^2 + y^2 \right)^{3/2}} \)

(c) \( \kappa = \frac{\| f''(x) \|}{\left( 1 + (f'(x))^2 \right)^{3/2}} \)

(d) \( A = \pi r^2 \)

**Solution**
(a) \( x = \frac{2}{(1 + 4x^2)^{3/2}} \)

(b) \( x(3) = \frac{2}{3^{3/2}} \approx 0.6088864317 \)
#4 Use the formula

\[
\tau = \frac{\begin{vmatrix} x & y & z \\ x & y & z \\ x & y & z \end{vmatrix}}{\|\vec{\nabla} \times \vec{a}\|^2}
\]

to find the torsion of \( \vec{r}(t) = \langle t^3, t^2, t \rangle \) at \( t=1 \)

\[\text{Sol:} \quad \begin{vmatrix} t^2 & t & 1 \\ 2t & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} = \begin{pmatrix} -2 \end{pmatrix}\]

\[\vec{\nabla} \times \vec{a} = \langle t^2, t, 1 \rangle \times \langle 2t, 1, 0 \rangle = \langle 0 - i, 2t - 0, t^2 - t^2 \rangle = \langle -1, 2t, -t^2 \rangle\]

\[\|\vec{\nabla} \times \vec{a}\|^2 = 1 + 4t^2 + t^4\]

\[\tau = \frac{-2}{1 + 4t^2 + t^4}\]

So \( \tau(1) = \frac{-2}{6} = \frac{-1}{3} \) \( \tau(1) = \frac{-1}{3} \)

#5 Given: \( \vec{r}(t) = \langle t, t^2, 1 \rangle \)

- Find \( \vec{r}'(t) \)
- Find \( \vec{r}'(1) \)

\[\text{Sol:} \quad \vec{r}'(t) = \langle 1, 2t, 0 \rangle \quad \text{and} \quad \|\vec{r}'(t)\| = (1 + 4t^2)^{1/2}\]

\[\vec{r}'(t) = \frac{1}{\|\vec{r}'(t)\|} \vec{r}'(t) = (1 + 4t^2)^{-1/2} \langle 1, 2t, 0 \rangle \]

\[\vec{r}'(1) = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \rangle\]