Tallahassee Community College  
MAC 2313=45707  
QUIZ LIMITS  
MR. JONES

LAST Key.  
FIRST D.  
DATE of QUIZ: Fri 10-13-2006

Show work in logical progression. Box your answers. Make an effort to write neatly.

#1. Show that \( \lim_{(x,y) \to (0,0)} \frac{xy + y^3}{x^2 + y^2} \) does not exist (DNE) by showing that limits as \( (x,y) \to (0,0) \) along different pathways yield different limits. Hint: Try pathway (a) along the x-axis, \( y = 0 \) compared to pathway (b) along the line \( y = 2x \).

\[
\begin{align*}
\text{Along } x\text{-axis: } y &= 0 \\
\lim_{(x,y) \to (0,0)} \frac{xy + y^3}{x^2 + y^2} &= \lim_{(x,0) \to (0,0)} \frac{0}{x^2} = 0 \\
\end{align*}
\]

\[
\begin{align*}
\text{Along } y &= 2x \\
\lim_{(x,y) \to (0,0)} \frac{xy + y^3}{x^2 + y^2} &= \lim_{(x,2x) \to (0,0)} \frac{2x^2 + 8x^3}{x^2 + 4x^2} \\
&= \lim_{(x,2x) \to (0,0)} \frac{x^2(2 + 8x)}{5x^2} = \lim_{(x,2x) \to (0,0)} \frac{2 + 8x}{5} = \frac{2}{5}
\end{align*}
\]

\[
\begin{align*}
\text{Along } y &= 0 \\
\lim_{(x,y) \to (0,0)} \frac{xy + y^3}{x^2 + y^2} &= \lim_{(x,0) \to (0,0)} \frac{xy + y^3}{x^2 + y^2} \\
&= \lim_{(x,0) \to (0,0)} \frac{xy + 0}{x^2 + 0} = \lim_{(x,0) \to (0,0)} \frac{xy}{x^2} \quad \text{DNE}
\end{align*}
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