Course Title: Calculus with Analytic Geometry III  
SCNS Number: MAC 2313,  
Prepared by: Russell J. Webster  
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**COURSE DESCRIPTION:**  
MAC 2313 CALCULUS WITH ANALYTIC GEOMETRY III (4) F, Sp, Sm. Prerequisites: Completion of MAC 2312 with a grade of “C” or better. Topics include vectors; equations of planes and lines in space; vector-valued functions, including unit tangent and unit normal vectors; velocity and acceleration of objects in space, and curvature; multivariable functions; the differential and integral calculus of multivariable functions; line and surface integrals, including Green’s Theorem, the Divergence Theorem, and Stoke’s Theorem. Use of a graphing calculator required. Please check with your instructor for the most appropriate one for you. Lecture 4 hours.

**GOALS OF THE COURSE:**  
This course is designed for mathematics, science, engineering, and mathematics education majors and for students interested in continuing a rigorous introduction to calculus concepts. This course can be used to satisfy part of the General Education Mathematics Requirement or it can serve as an elective.

**PERFORMANCE OBJECTIVES:**  
By the end of this course the student should be able to ...

[Note: In the following "state" means "quote" or "paraphrase accurately."]

**Vector-Valued Functions**  
1) define vector-valued functions and component functions.  
2) sketch the curve described by a vector-valued function.  
3) perform addition, subtraction, dot products, and cross products of two vector-valued functions.  
4) perform multiplication and composition of a scalar-valued function and a vector-valued function.  
5) define, explain, and use the concepts of limits, differentiation, integration, and continuity of vector-valued functions.  
6) prove and use the basic theorems (rules) for differentiability of vector-valued functions, including the sum, difference, cross product, and dot product of two
vector-valued functions; and the product and composition of a scalar-valued function and a vector-valued function.

7) define, explain, and use the unit tangent vector and the unit normal vector.
8) define, explain, and use the length of a smooth curve which is described by a vector-valued function.
9) define and explain the concepts of velocity, speed, and acceleration of an object moving through 3-space along a path described by a vector-valued function.
10) define and explain the concepts of tangential and centripetal components of acceleration.
11) use objectives 9) and 10) to solve a wide variety of problems, including many applications problems.
12) define, explain and use the concepts of curvature vector, curvature, and radius of curvature at a point.
13) be familiar with the use of vector-valued function techniques and Newton's Second Law of Motion and his Law of Gravitation to derive Kepler's Laws of Planetary Motion. (This section is optional for detailed in-class coverage).

Polar Coordinates and Parametric Equations
1) convert between Cartesian and polar systems (both ways).
2) graph polar equations.
3) compute areas of regions using polar coordinates.
4) graph parametric equations.
5) find equations of tangent lines to curves which are given by parametric equations.
6) define and compute the arc length of a curve C which is described parametrically.

Multivariable Functions
1) define the concept of a real-valued function of n real variables.
2) evaluate real-valued functions of several real variables and determine their domains.
3) sketch the graph of \( z = f(x,y) \) in \( \mathbb{R}^3 \) using level curves if necessary.
4) sketch the graph of a level surface of \( f: w = f(x,y,z) \) in \( \mathbb{R}_3 \) where \( w \) is a real-valued function of 3 real variables.
5) recognize and graph the standard quadric surfaces: ellipsoids, elliptic hyperboloids of one sheet or two sheets, elliptic paraboloids, hyperbolic paraboloids, and the degenerate surfaces: elliptic cylinders, elliptic cones, parabolic cylinders, hyperbolic cylinders, intersecting planes, and parallel planes.
6) define, explain, and use the cylindrical coordinate system in 3-space.
7) define, explain, and use the spherical coordinate system in 3-space.
8) convert coordinates of a point in 3-space from any one of the three systems: Cartesian, Cylindrical, or Spherical, to any other of the three systems.
9) transform the equations in one system which describe a given region into equations in another system (for the 3 systems Cartesian, Cylindrical, Spherical).
10) sketch regions in objective 9).
The Differential Calculus of Multivariable Functions

1) define, explain, and use the limits of functions \( z = f(x,y) \) of two variables and \( w = f(x,y,z) \) of three variables.

2) define and use the concept of the continuity of a function \( z = f(x,y) \) at a point \((a,b)\).

3) extend the theorem on sums, differences, products, and quotients of limits from functions of one variable to functions of two variables.

4) extend the theorem on the continuity of sums, differences, products, and quotients of continuous functions from single-variable to multivariable functions.

5) use the theorem on the limit of a composition \( g \circ f \) where \( f \) is a real-valued function of two variables and \( g \) is a function of one variable.

6) define and use the concept of continuity on a region of functions of two or three variables.

7) define and use first and second partial derivatives.

8) define differentiability of a function \( z = f(x,y) \) at a point.

9) prove and use the theorem: \( z = f(x,y) \) differentiable at a point \( \Rightarrow \) \( f \) is continuous at that point.

10) prove and use the theorem: If the partial derivatives of \( z = f(x,y) \) are continuous at a point, then \( f \) is differentiable at that point.

11) perform calculations involving differentials.

12) prove and use the chain rule for functions of two variables.

13) prove and use: \( F(x,y) = 0 \) defines implicitly \( y \) as a function of \( x \)

\[ \frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)} \]

14) define, explain, and use directional derivatives and gradients for functions of 2 or 3 real variables.

15) use the technique of finding the gradient to a level surface to determine the tangent plane.

16) define, explain, and use absolute (relative) maxima and minima.

17) state and use the Extreme Value Theorem for functions of 2 or 3 real variables.

18) define, explain, and use critical points for functions of two variables.

19) determine when a function of two variables has a relative maximum, relative minimum, or a saddle point at \((x_0, y_0)\) based upon a consideration of \( f_{xx}(x_0,y_0) \) and \( D(x_0,y_0) = f_{xx}(x_0,y_0) f_{yy}(x_0,y_0) - [f_{xy}(x_0,y_0)]^2 \).

20) determine the extrema of a function subject to one or more constraints using Lagrange multipliers.

Integral Calculus of Multivariable Functions

1) develop, explain, and apply the concept of the double integral of \( f \) over \( R \) as the limit of Riemann sums:

2) define volume as a double integral and use this to find volumes bounded by closed and bounded regions \( R \) in the \( XY \)-plane and surfaces \( z = f(x,y) \).

3) use iterated integrals to evaluate double integrals.
4) develop, explain, and apply the concept of the double integral in polar coordinates.
5) define, explain, and use the mass of a lamina S as a double integral, the moments of S about the X-axis and about the Y-axis, the center of mass of S, the moments of inertia of S about the X-axis and about the Y-axis, and the radii of gyration about the X-axis and about the Y-axis.
6) define, explain, and use the concept of surface area of a smooth surface as a double integral.
7) repeat objective 1) for triple integrals.
8) repeat objective 3) for triple integrals.
9) find volumes of solid regions using triple integrals.
10) transform triple integrals from rectangular to polar or cylindrical coordinates in order to facilitate evaluation.
11) transform equations from Cartesian (rectangular) to either polar or cylindrical coordinates in order to have relatively straightforward limits of integration for iterated triple integrals.
12) apply triple integration to solve a wide variety of applications problems, including centroids, radii of gyration, and average value problems.

Line and Surface Integrals
1) define, explain, and use the divergence and curl of a differentiable vector field.
2) apply the standard results regarding both the algebra of the divergence and curl and the interrelationships between the divergence and the curl.
3) develop, explain and apply the concepts of line integrals in the X-Y-, and Z-coordinate directions and with respect to arc length.
4) apply line integrals to solve problems involving work and moments.
5) prove and explain the theorem linking "independence of path" with "conservative integrand."
6) use 5) to establish the Fundamental Theorem of Line Integrals.
7) develop and use necessary and sufficient conditions for a vector field's being conservative in an open, simply connected region.
8) state, prove, and use Green's Theorem.
9) perform change of variables transformations of double integrals using Jacobians.
10) define, explain, and use the concept of surface integral.
11) define, explain, and use the concept of an orientable smooth surface.
12) state and use (proof optional) the Divergence Theorem.
13) state and use (proof optional) Stoke's Theorem.
14) apply the Divergence Theorem and Stoke's Theorem to solve problems involving fluid flow, heat flow, and Maxwell's equations.