Problem 1: Rewrite the integral in the order
\[ \boxed{\text{a) } dy \\ dz \\ dx \\ b) dy \\ dx \\ dz \\ c) dx \\ dy \\ dz \\ d) dx \\ dz \\ dy} \]

Given \[ \int_{x=-1}^{x=1} \int_{y=x^2}^{y=1} \int_{z=0}^{z=1-y} dz \\ dy \\ dx \]

The solid is defined by:
- Top: \( y + z = 1 \)
- Side: \( y = x^2 \)
- Bottom: \( z = 0 \)

Solution:
1. I want to look at some different "views."

   - \(xy\)-plane: \( y = x^2 \), \( x = \pm \sqrt{y} \)
   - \(yz\)-plane: \( y = 1 - z \), \( y = 1 - z \)
   - \(xz\)-plane: \( z = 1 - x^2 \), \( x = \pm \sqrt{1 - z} \)

2. \(dy \\ dz \\ dx\) For \( x \) and \( z \) fixed, \( y \) varies from \( y = x^2 \) to \( y = 1 - z \).
   For \( x \) fixed, \( z \) varies from \( z = 0 \) to \( z = 1 - x^2 \).
   \( x \) varies from \( x = -1 \) to \( x = 1 \)

\[ \int_{x=-1}^{x=1} \int_{z=0}^{z=1-x^2} \int_{y=x^2}^{y=1-z} dy \\ dz \\ dx \]

Cont...
b) \[ \frac{dy}{dx} dz \]

With \( z \) and \( x \) fixed, \( y \) varies from \( y = x^z \) to \( y = 1 - z \).

With \( z \) fixed, \( x \) varies from \( x = \sqrt{1 - z} \) to \( x = \sqrt{1 - z^2} \)

(see fig. \( \theta \))

\[ \int_{z=0}^{z=1} \int_{x=-\sqrt{1-z}}^{x=\sqrt{1-z^2}} \int_{y=x^2}^{y=1-z} dy \, dx \, dz \]

- \[ dx \, dy \, dz \] with \( z \) and \( y \) fixed, \( x \) varies from \( x = -\sqrt{y} \) to \( x = \sqrt{y} \)

(see fig. \( \theta \))

With \( z \) fixed, \( y \) varies from \( y = 0 \) to \( y = 1 - z \)

\( z \) varies from \( z = 0 \) to \( z = 1 \)

\[ \int_{z=0}^{z=1} \int_{y=0}^{1-z} \int_{x=-\sqrt{y}}^{x=\sqrt{y}} dx \, dy \, dz \]

Etc., etc., etc.

- \[ \#2 \] §15.4: p. 1092: #41 Change the order of integration (in an appropriate way) and integrate, \( I = \int_{z=0}^{z=1} \int_{y=0}^{1-z} \int_{x=2y}^{x=2} \frac{4 \cos(x^2)}{2 \sqrt{y}} \, dx \, dy \, dz \)

Solv.

\[ I = 2 \left( \int_{z=0}^{z=1} \frac{z^{1/2}}{2} \, dz \right) \left( \int_{y=0}^{1-z} \int_{x=2y}^{x=2} \cos(x^2) \, dx \, dy \right) = 2 \left( \frac{z^{1/2}}{2} \right)_{z=0}^{z=1} \left( \int_{y=0}^{1-z} \int_{x=0}^{x=2} \cos(x^2) \, dx \, dy \right) \]

\[ = 4 \cdot 2 \int_{x=0}^{x=2} \frac{1}{2} x \cos(x^2) \, dx \]

Let \( u = x^2 \), \( du = 2x \, dx \)

\[ = 0, u = 0 \quad x = 2, u = 4 \]

\[ = 2 \int_{u=0}^{u=4} \cos(u) \, du = 2 \{ \sin(u) \} \bigg|_{u=0}^{u=4} = 2 \sin(4) \]