\[ \vec{F} = \langle e^{y+2z}, xe^{y+2z}, 2xe^{y+2z} \rangle \]

**Find \( f \) such that \( \nabla f = \vec{F} \)**

**Solution**

1. Check for conservative.

\[
\begin{align*}
\frac{\partial M}{\partial y} &= e^{y+2z} & \frac{\partial N}{\partial z} &= e^{y+2z} & \checkmark \\
\frac{\partial M}{\partial z} &= 2xe^{y+2z} & \frac{\partial P}{\partial x} &= 2xe^{y+2z} & \checkmark \\
\frac{\partial N}{\partial x} &= 2xe^{y+2z} & \frac{\partial P}{\partial y} &= 2xe^{y+2z} & \checkmark
\end{align*}
\]

\( \therefore \vec{F} \) is conservative

2. \( M = \frac{\partial f}{\partial x} \)

\( f = \int M \, dx = \int e^{y+2z} \, dx \)

\[
\begin{align*}
f &= xe^{y+2z} + g(y,z)
\end{align*}
\]

3. \( N = \frac{\partial f}{\partial y} = xe^{y+2z} + \frac{\partial g}{\partial y} \equiv xe^{y+2z} \)

Conclusion: \( \frac{\partial g}{\partial y} = 0 \)

\( \therefore g(y,z) = g(z) \)

\[
\begin{align*}
f &= xe^{y+2z} + g(z)
\end{align*}
\]

4. \( P = \frac{\partial f}{\partial z} = 2xe^{y+2z} + \frac{\partial g}{\partial z} \equiv 2xe^{y+2z} \)

\( \therefore \frac{\partial g}{\partial z} = 0 \)

\( \therefore g(z) = C \)

\[
\begin{align*}
f(x,y,z) &= xe^{y+2z} + C
\end{align*}
\]