Solve the IVP,

\[(1+4y^2)dy + x(1+4y^2)dx = 0\]

\[y(1) = 0\]

**Solution:**

1. To separate the variables, divide both sides of the ODE by \((1+4y^2)(1+4y^2)\), obtaining

\[\frac{dy}{1+4y^2} + \frac{x}{1+x^4} = 0\]

\[(**)\]

2. Now let's integrate each of the terms of \((**)*\) separately, and then put the pieces together.

- **a)** \[\int \frac{dy}{1+4y^2}\]
  
  Let \(u = 2y\), so \(du = 2dy\)
  
  and \(\frac{1}{2} du = dy\)
  
  and \(u^2 = 4y^2\)

  \[= \frac{1}{2} \arctan(u) + C = \frac{1}{2} \arctan(2y) + C\]

- **b)** \[\int \frac{x}{1+x^4} dx\]
  
  Let \(u = x^2\), so \(du = 2x dx\)
  
  and \(\frac{1}{2} du = x dx\)
  
  and \(u^2 = x^4\)

  \[= \frac{1}{2} \arctan(u) + C = \frac{1}{2} \arctan(x^2) + C\]

3. Thus from \((**)*\) we get (combining all the "C's")

\[\frac{1}{2} \arctan(2y) + \frac{1}{2} \arctan(x^2) = C\]

or \[\arctan(2y) + \arctan(x^2) = C\] (This "new" \(C\) is twice the "old" \(C\)).

4. Now we look at the IV, \(y(1) = 0\).

Thus \(x = 1\) \& \(y = 0\). Subs. into \((***)\)

\[C = \arctan(0) + \arctan(1)\]

\[= 0 + \frac{\pi}{4}\]

\[\arctan(2y) + \arctan(x^2) = \frac{\pi}{4}\]

is the solution to the IVP.