Problems #1, 4, 5, and 8 are 12 points each. Problems 2, 3, 6, and 7 are 13 points each. The Bonus is 10 points.

#1/ Verify that \( y_1 = 2e^{-4x} \) is a solution of \( y'' + 3y' - 4y = 0 \).

#2/ State explicitly the method you are using and solve: \( y\,dx + (1 + x)\,dy = 0 \). Assume \( x > 0 \) and \( y > 0 \).

#3/ Initially 80 lbs. of salt are dissolved into a large tank holding 400 gal. of water. A brine solution is pumped into the tank at the rate of 2 gal per min., and the well-stirred solution is then pumped out at the same rate. If the concentration of the salt solution entering is 0.5 pounds per gallon, \( [A] \) determine the amount of salt in the tank at time \( t \). Also, \( [B] \) approximately how much salt is in the tank after ten hours? \( \text{[For part [B] give complete approximate calculator answer and then round your final answer to the nearest pound. Write sentence for final answer.]}\)

#4/ Verify that the operator \( (D - 1)(D + 3) \) annihilates the function \( 4e^x + 5e^{-3x} \).

#5/ Are these functions linearly dependent or linearly independent? Show why.
\[
f_1(x) = 3x - 4, \quad f_2(x) = 4x - 8, \quad f_3(x) = 10x - 13.
\]

#6/ A 16-lb weight attached to a spring exhibits simple harmonic motion. There is no damping. Determine the equation of motion if the spring constant is 4 lb/ft and if the weight is released 2 ft below the equilibrium position with a downward velocity of 1 ft/sec. (Assume \( g = 32 \text{ ft/sec}^2 \)).

#7/ Solve this IVP using Laplace Transforms.
\[
y'' + 4y' + 4y = e^{-2t} \quad \text{and} \quad y(0) = 0 \quad \text{and} \quad y'(0) = 0.
\]

#8/ Find \( [\text{A}] \) \( \mathcal{L}\{f(t)\} \) if \( f(t) = 4 - e^{-3t} \sin t + \cos 2t \) \( [\text{B}] \) \( \mathcal{L}^{-1}\{F(s)\} \) if \( F(s) = \frac{s + 2}{s^2 + 16} \)

#9(bonus) // Re-write the given piecewise function in terms of unit step functions.
\[
f(t) = \begin{cases} 
0 & \text{if } 0 \leq t < 1 \\
t^2 + 2 & \text{if } 1 \leq t < 5 \\
0 & \text{if } 5 \leq t
\end{cases}
\]