ODE DREAMIN’
CHAPTER 1
OPENING REMARKS (5-02-07 & rev. 5-05-10) by Doug Jones

\[ \frac{dy}{dx} + 5y = 0 \] (1.1)

is a differential equation, because it is an equation with a derivative in it! It’s actually an ordinary differential equation (that’s where “ODE” comes from).

A solution to (1.1) is a function, \( f(x) \), with the property that if I “plug it into” the LHS (left-hand side) of (1.1) and “do the math,” it simplifies down to the RHS (right-hand side), which in this problem is zero.

Many times, especially in the early-going, I’ll write \( \phi(x) \) instead of \( f(x) \) for a solution, so don’t let this notation surprise you.

My claim, now, is that

\[ \phi(x) = e^{-5x} \] (1.2)

is a solution to (1.1).

How did I come up with that function? – That’s one big question! That’s the meat of this course, and we’ll partially answer that question during the next 10 weeks. However, the short answer is that we use the “McBee Method” – we ask it! That is, we ask the differential equation, “What is your solution?” And then we listen very carefully. Many times it will answer us! . . . . But enough of that for now; we’ll get into that later.

How do I know that (1.2) is in fact a solution to (1.1)? That’s another very important question, and in many cases in “Intro DE” courses, it’s not too hard to answer. We answer by a process called Verification. That is, we verify that \( \phi(x) \) “solves” (1.1).

And just how do we do this? Simple. We just “plug” \( \phi(x) \) into one side (in this case, the left) of (1.1) and “chug” it out! – Note: “plug and chug” is a euphemism for “substitute and simplify.”

Here’s how it writes-up:

Verify that \( \phi(x) = e^{-5x} \) is a solution of \( \frac{dy}{dx} + 5y = 0 \).

Verification: 

[1] \( \phi(x) = e^{-5x} \), so \( \phi'(x) = -5e^{-5x} \).

[2] “Plug and chug.” – Substituting these values into the LHS of the differential equation and going through successive steps of simplification, we get

\[ \frac{d}{dx} \phi(x) + 5\phi(x) = -5e^{-5x} + 5e^{-5x} = 0 \] (which is the RHS of (1.1)).

So this is the basic style, format, or look that one expects to see in the verification of a solution. There are a few variations on this theme, and we’ll “cross those bridges” when we get to them.