MAP 2302 - 79123  

**PROBLEM REQUEST**

8.2.3: p. 45: # 25. Solve using sub.

\[ y \frac{dx}{dy} = x + 4ye^{-2x/y} \quad (*) \]

**Solution**

**1.** Put \((*)\) into \(Mdx + Ndy = 0\) form, if possible.

\[ ydx + (-x - 4ye^{-2x/y})dy = 0 \quad (**) \]

**2.** If Not Required - Check for "homogeneous of deg. n."

\[ M(tx, ty) = ty = tM(x, y) \quad \therefore M \text{ is homogeneous of deg 1.} \]

\[ N(tx, ty) = (-tx - 4tye^{-2tx/ty}) = t(-x - 4ye^{-2x/y}) = tN(x, y) \]

\[ \therefore N \text{ is also homogeneous of deg 1.} \]

\[ \therefore (\ast) \text{ is homogeneous of deg 1.} \]

\[ \therefore (\ast) \text{ can be solved via a subs. } \]

**3.** Let \(x = vy\), \[ dx = vdy + ydv. \] Subs into \((**):\]

\[ y(vdy + ydv) - (vy + 4ye^{-2vy/y})dy = 0 \quad (***) \]

Require \(y \neq 0\) and divide b.s. of \((***)\) by \(y:\]

\[ vdy + ydv - ydy - 4e^{-2v}dy = 0 \quad (***) \]
\[ y \, dv - 4e^{-2v} \, dy = 0 \] (***)

4. Separate the variables in (***), and integrate.

\[ \frac{e^{2v}}{4} \, dv - \frac{dy}{y} = 0 \] (variables ARE separated).

\[ \frac{e^{2v}}{8} - \ln y = C \] (Require \( y > 0 \))

\( \frac{e^{2v}}{8} \) \( \text{OK Ans.} \)

5. We can go on:

\[ \frac{e^{2v}}{8} = \ln y + \ln C \] ("new C")

\[ = \ln Cy \]

\[ :e^{2v} = 8 \ln Cy \]

\[ e^{2x/y} = 8 \ln Cy \] \( \text{OK Ans} \)