20100526 W 2231 @ HM

ODE@TCC // MAP2302 - 79123 // SUMMER’10

REQUEST PROBLEM

53.2: p.91: # 25 A large tank is partially filled w/ 100 gal of fluid in which 10 lb of salt is dissolved. Brine containing 1/2 lb. salt per gal. the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a slower rate of 4 gal/min. Find the number of pounds of salt in the tank after 30 min.

\[ \text{Rate In: } 6 \text{ gal/min} \times \frac{1}{2} \text{ lb salt}{\text{gal}} = 3 \text{ lb/min} \]

\[ \text{Rate Out: } 4 \text{ gal/min} \times \frac{A}{100 + 2t} \text{ lb salt}{\text{gal}} = \frac{4A}{100 + 2t} \text{ lb/min} \]

Let me explain the \( \frac{A}{100 + 2t} \) lb salt/gal.

First, this is the concentration of salt in the outflow. There are \( A = A(t) \) lb of salt in the tank at time \( t \) — we just don’t know specifically what \( A \) is right now — In fact that is precisely what we are trying to find out.

Second, fluid is coming in @ 6 gpm and going out at 4 gpm. Thus the volume in the tank is increasing at the rate of 2 gpm. Thus at time \( t \) there are \( 100 + 2t \) gal in the tank.

Now we can get on w/ the math.

(over)
\[
\frac{dA}{dt} = \text{Rate IN} - \text{Rate OUT.}
\]

\[
\frac{dA}{dt} = 3 - \frac{4A}{100+2t} = 3 - \frac{A}{25+0.5t}
\]

\[
\frac{dA}{dt} + \frac{A}{25+0.5t} = 3 \quad (\star) \quad \text{Std Form.}
\]

4. \text{IF:} \quad P(t) = \frac{1}{25+0.5t} = \frac{2}{50+t}

\[
I(t) = \int \frac{2}{50+t} \, dt = 2 \ln |t+50| = \ln (t+50)^2
\]

\[
\mu(t) = e^{I(t)} = (t+50)^2
\]

5. \text{Multiply & Compress:}

\[
(t+50)^2 \frac{dA}{dt} + (t+50)^2 \cdot \frac{2}{t+50} A = (t+50)^2 \cdot 3
\]

\[
(t+50)^2 \frac{dA}{dt} + 2(t+50) A = 3(t+50)^2
\]

(compress): \[
\left[ (t+50)^2 A \right]' = 3 (t+50)^2
\]

6. \text{Integrate:}

\[
(t+50)^2 A = 3 \int (t+50)^2 \, dt
\]

\[
= (t+50)^3 + C
\]

7. \text{Solve:}

\[
A = (t+50) + C (t+50)^{-2} \quad (\star\star)
\]

(cont. 7)
\[ A(0) = 10 \text{ lb salt.} \]

Find \( C \):

\[ 10 = A(0) = (0 + 50) + C(0 + 50)^{-2} \]
\[ 10 = 50 + \frac{C}{2500} \implies -40 = \frac{C}{2500} \implies C = -100000 \]

\[ A(t) = (t + 50) - \frac{100000}{(t + 50)^2} \text{ lb salt.} \]

At the end of 30 min, there are approximately 64.4 lb salt in the tank.

Work: \[ A(30) = 80 - \frac{100000}{80^2} = 80 - \frac{100000}{6400} \]
\[ = 80 - \frac{250}{16} = 80 - \frac{125}{8} = \frac{640 - 125}{8} \]
\[ = \frac{515}{8} = 64.375 \]