I love questions — some have answers; some don’t. Here’s a question with an answer.

RE: §4.1.2: p. 130: #15.

To get insight into matrix-reduction techniques.

"Determine whether the given functions are linearly independent or dependent on \((-\infty, \infty)\)."

\[ f_1(x) = x, \quad f_2(x) = x^2, \quad f_3(x) = 4x - 3x^2 \]

**Solution:**

1. Set LC equal to zero.

\[ c_1 x + c_2 x^2 + c_3 (4x - 3x^2) = 0 \quad (*) \]

2. Put into std. polynomial form on LHS: \((c_2 - 3c_3)x^2 + (c_1 + 4c_3)x = 0\)

3. "Extract" system:

\[
\begin{align*}
 c_2 - 3c_3 &= 0 \\
 c_1 + 4c_3 &= 0 \\
 c_1 + c_2 + c_3 &= 0
\end{align*}
\]

\[ (***) \]

Note: in Eqn. (***) , we can consider it as

\[ (c_2 - 3c_3)x^2 + (c_1 + 4c_3)x + (c_1 + c_2 + c_3) = 0 \]

4. Set up Augmented Matrix

\[
\begin{bmatrix}
0 & 1 & -3 & | & 0 \\
1 & 0 & 4 & | & 0 \\
1 & 1 & 1 & | & 0
\end{bmatrix}
\]

5. Enter in TI-84: run rref(.

\[
\begin{bmatrix}
0 & 1 & -3 & | & 0 \\
1 & 0 & 4 & | & 0 \\
1 & 1 & 1 & | & 0
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & 0 & 4 & | & 0 \\
0 & 1 & -3 & | & 0 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

6. \[ c_1 + 4c_3 = 0 \]

\[ c_2 - 3c_3 = 0 \]

\[ c_1 = -4c_3 \]

\[ c_2 = 3c_3 \]
Note: step 6 re-interprets the augmented system of equations equivalent to (***)

7. Let \( c_3 \) be any number and you have a "triple" for \( c_1, c_2, c_3 \).

Now it proves nothing if we let \( c_3 = 0 \), and it is a WOT (waste of time) to let \( c_3 = \sqrt{2} \), or \( i \), or 2010.

Let \( c_3 = 1 \), and it follows that \( c_1 = -4 \) and \( c_2 = 3 \) and -- don't leave out the last step...

8. Display the conclusion:

The functions \( f_1(x) = x, f_2(x) = x^2, f_3(x) = 4x - 3x^2 \) are LD (linearly dependent) on \((-\infty, \infty)\)
because \(-4f_1(x) + 3f_2(x) + 1f_3(x) = 0\) on \((-\infty, \infty)\)

\[-4x + 3x^2 + 1(4x - 3x^2) = -4x + 3x^2 + 4x - 3x^2 = 0\]