
\[ y''' + 3y'' + 3y' + y = 0 \quad (\ast) \]

**Soln:**

1. **AuxEq:** \[ m^3 + 3m^2 + 3m + 1 = 0 \quad (\ast\ast) \]

2. **Solve AuxEq:**

---

**Note:** The LHS of \((\ast\ast)\) factors into \((m+1)^3\), but in case you don't see that, we always have synthetic division (remember your PreCalc?) or plain long division. So let's say we don't see the factorization...

**SD (Synthetic Division) - Rational Zeros Thm.** If

\[ a_n m^n + a_{n-1} m^{n-1} + ... + a_1 m + a_0 \]

is a poly in "m",

then any rational zero of the poly. must be of the form

\[ \pm \frac{\text{a factor of } a_0}{\text{a factor of } a_n} \]

So in THIS problem \( a_n = a_3 = 1 \) and \( a_0 = 1 \).

So any rational root of \((\ast\ast)\) must be of the form \( \pm \frac{1}{1} = \pm 1 \)

Now I know that \( \pm 1 \) won't work b/c all the signs in \((\ast\ast)\) are positive and I can't have a bunch of positive things adding-up to zero! So I'll try \( -1 \).

**Work:**

\[
\begin{array}{c|cccc}
-1 & 1 & 3 & 3 & 1 \\
0 & -1 & -2 & -1 \\
\hline
1 & 2 & 1 & 0
\end{array}
\]

\[ \text{Remainder is zero} \]

\[ \therefore \text{ \((\ast\ast)\) Factors: } (m+1)(m^2 + 2m + 1) = 0 \]

\[ \therefore \quad (m+1)(m+1)(m+1) = 0 \quad (\text{cont...}) \]
\[
(m + 1)^3 = 0
\]

\[(* * *)\]

And \( m = -1 \) w/ a multiplicity of 3.

\[
3 \text{ Gen Sol: } y = c_1 e^{-x} + c_2 xe^{-x} + c_3 x^2 e^{-x}
\]

\( END. \)