§4.3: p.145: #59: Find the Gen Sol if we know
\[ y_1 = e^x \]
\[ y''' - 9y'' + 25y' - 17y = 0 \]  \( (*) \)

**Solution:**

1. Aux Eq:
\[ m^3 - 9m^2 + 25m - 17 = 0 \]  \( (**) \)

2. Now, since \( y_1 = e^x \) is a sol?, and a quick mental check verifies this.

   this means that \( m_1 = 1 \) (i.e. \((m-1)\) is a factor of \( (**) \). Thus, we should be able to "divide it out."

\[
\begin{array}{c|cccc}
    & 1 & -9 & 25 & -17 \\
0 & -8 & 17 & 0 & & \checkmark
\end{array}
\]

\[ \therefore \ (**) \text{ factors thus :} \]
\[ (m-1)(m^2-8m+17) = 0 \]

and so to find the "other" values of \( m \), we must solve
\[ m^2 - 8m + 17 = 0 \]
\[ m^2 - 8m + 16 = -1 \]
\[ (m-4)^2 = -1 \]
\[ m = 4 \pm i \]
\[ m = 4 \pm i \quad \alpha = 4, \beta = 1 \]

3. Gen Sol:
\[ y = c_1 e^x + c_2 e^{4x} \cos x + c_3 e^{4x} \sin x \]

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\text{ Note: After I finished this there was a thunderstorm going, so I couldn't run my computer. And I spent some time working out the Wronskian for these 3 functions. The Wronskian is never zero. Thus the 3 functions ARE LI. But you don't have to do this -- it took me 2 full pages and 1/2 hr. I did it just for fun. } \]