INSTRUCTIONS: Do all work and box your answers on the “worksheets.” Do no more than 3 problems per page. Draw a horizontal line segment between problems. Do not write in the margins. Do not write on the back of any page. Write your name only on this sheet. Box your answers where practicable.

1. (16 pts) The following ODE is separable. Solve it using the method of separation of variables.

\[ 2 \frac{dy}{dx} = \frac{y(x+1)}{x} \]

Your answer may be implicit. 3 points bonus for a (correct) explicit solution.

2. (17 pts) Given that \( y_1 = e^{4x} \) is a solution to \( y'' - 3y' - 4y = 0 \) on \( -\infty < x < \infty \), use the method of Reduction of Order (RO) to determine another solution.

3. (17 pts) Given:

\[ 2xy - 9x^2 + (2y + x^2 + 5) \frac{dy}{dx} = 0 \]

[A] Determine if this ODE is exact. Show your work. If it is not exact, then you are finished with this problem. If it is exact, then...

[B] Solve it using the method of exact differential equations.

4. (17 pts) Solve the following linear IVP by applying an IF (Integrating Factor) to the given LODE.

\[ \frac{dy}{dx} - 3y = x ; \quad y(0) = 1. \]

5. (17 pts) A tank contains 800 liters of brine in which 100 g of salt is dissolved. Brine containing 3 g of salt per liter is then pumped into the tank at a rate of 10 liters per minute; the well-mixed solution is pumped out at the same rate.

a. Find the number of grams of salt \( A(t) \) in the tank at time \( t \).

b. How much salt is in the tank at the end of one hour? (Give first the complete calculator answer – to six decimal places, and then round off your final approximation to one decimal place. Write a complete sentence for your final answer to this part.)

6. (16 pts) Determine whether the given functions are linearly independent or dependent on \( (-\infty, \infty) \):

\[ f_1(x) = 4x - 2, \quad f_2(x) = 2x, \quad f_3(x) = 6 \]

[ BIG HINT: Don’t even think about using a Wronskian here! ]

7. (BONUS – 10 points) Verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. \( y'' - y = 0, \quad e^x, \quad e^{-x}, \quad ( -\infty, \infty ) \)

[ You have two things to verify: (i) that each function is a solution and (ii) that they are linearly independent. (Use Wronskian here.) ]