There are 4 problems @ 17 points each and 2 problems @16 points each. There is one bonus problem at 10 points. Do no more than two problems per page. Draw a horizontal line segment between problems. Box answers. Leave left & top margins clear – except for page numbering. Turn in all sheets, including scrap paper.

#1 (16 pts) Verify that $\phi(t) = e^{-\frac{t}{2}}$ is a solution to $2\frac{dx}{dt} + x = 0$. (Note: x is a function of t.)

#2. (16 pts) State method of solution (or type of equation) and solve. You may box implicit solution.

\[ \frac{dy}{2 + x} - y^2 \, dx = 0 \]

#3. (17 pts) A large tank has 500 gal. of brine in which there is 20 lb. salt dissolved. Liquid is pumped into the tank at the rate of 10 gpm (gal per min). The input contains 2 lb salt per gal. The well-mixed mixture is pumped out of the tank at the same rate. [A] Derive the model for the amount, $A$ (lbs.), of salt in the tank at any time, $t$ (min.). Then [B] determine the amount of salt in the tank at the end of one hour. Give the full calculator answer (approximation). Then round your final answer (another approximation) off to the nearest lb. Write your final answer in sentence form. (For 3 extra points: What appears to be the maximum amount of salt that the tank can contain under this process. Justify your answer.)

#4. (17 pts) [A] Verify that this equation is exact. Then [B] solve this IVP using the exact method.

\[ 2xy \, dx + (x^2 - 1) \, dy = 0 , \quad y(0) = 3 \]

#5. (17 pts) Find the General Solution (in explicit form) of $\frac{dy}{dx} + y = e^{3x}$.

#6. (17 pts) Show that this equation is homogeneous in the sense of Euler. Then solve it using the techniques of §2.3: $(x + y) \, dx + x \, dy = 0$ You may report your answer in implicit form, but it must be in x’s and y’s.

Bonus (10 pts) Solve the Bernoulli equation. $x \frac{dy}{dx} + y = y^3$. You may report your answer in implicit form, but it must be in x’s and y’s.