TALLAHASSEE COMMUNITY COLLEGE

CALC II-MAT 2312-36686 - FALL 2005
TEST #4 (PT 1a) - Sects. 11.1, 11.2, 11.3, 11.4, 12.1, 12.2

TEXT: STEWART'S 5TH
INSTRUCTOR: D. JONES
DAY/DUE MON., 12/05/05

INSTRUCTIONS: Do this problem on these two pages. No calculator is allowed. When you are finished with this problem, turn it in, get out your calculator, and get back (from me) the other 4 problems on part #1 of this test.

Box your final answer and circle any "partial credit results."

Write on one side of this paper and do not cross the left margin,

#1/10. This is a NON-CALCULATOR problem. Also, please use a straight edge in constructing your graph.

[A]

Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

Eliminate the parameter to find a Cartesian equation of the curve.

$x = 5 - t, \quad y = -(t^2 - 4), \quad -4 \leq t \leq 4.$

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x = 5 - t$</th>
<th>$y = -(t^2 - 4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>9</td>
<td>-12</td>
</tr>
<tr>
<td>-3</td>
<td>8</td>
<td>-5</td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>-12</td>
</tr>
</tbody>
</table>

[B] Eliminate the parameter:

\[
\begin{align*}
0 \quad & x = 5 - t, \quad y = -(t^2 - 4), \quad -4 \leq t \leq 4 \\
1 \quad & x = 5 - t, \quad y = -(t^2 - 4), \quad -4 \leq t \leq 4 \\
& y = -(5 - x)^2 + 4 \quad \text{BETTER} \\
& y = -(x^2 - 10x + 25) + 4 \quad \text{OK.}
\end{align*}
\]

When you have finished this part of TEST #4 - Part #1, you need to "trade it in" for the rest of Part #1.
# 3/10 Find the distance between the points with polar coordinates $P_1(3, \frac{\pi}{3})$ and $P_2(1, \frac{\pi}{4})$.

\[ \begin{align*}
\sin^2 \theta & = \frac{1}{4} \\
\sin \theta & = \frac{1}{2} \\
\theta & = \frac{\pi}{6} \\
\end{align*} \]

\[ d(P_1P_2) = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)} = \sqrt{10 - 18 \cos \left( \frac{\pi}{6} \right)} \]

\[ = \sqrt{10 - 18 \left( \frac{\sqrt{3}}{2} \right)} \]

\[ = \sqrt{10 - 9} = \sqrt{1} \]

\[ \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} = \frac{9\pi}{8}, \frac{\pi}{3} \]

# 4/10 Find the area of the region enclosed by one loop of the curve $r = 2 \cos 4\theta$.

**Details:**

\[ \theta = 0 \Rightarrow r = 2, 4\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{8} \Rightarrow r = 0 \]

\[ \theta = \frac{\pi}{2}, 4\theta = \pi \Rightarrow r = -2, 2\theta = \frac{\pi}{2}, 4\theta = \frac{3\pi}{2} \Rightarrow r = 0 \]

\[ \theta = \frac{3\pi}{8}, 4\theta = \frac{3\pi}{2}, r = 2 \]

\[ \text{Area of one loop} = 2 \times \text{Area}_{\theta = \frac{\pi}{8}} \]

\[ A = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \int^{2 \cos 4\theta} \frac{1}{2} r^2 d\theta d\phi \]

\[ = \left\{ \int^{2 \cos 4\theta} \frac{1}{2} \left( 1 + \cos(4\theta) \right) d\theta \right\} \left\{ \int^{\frac{3\pi}{8}} \left[ \sin\theta \right]_{\theta = 0}^{\theta = \frac{3\pi}{8}} \right\} \]

\[ = \left\{ \int_{\theta = 0}^{\frac{3\pi}{8}} \frac{1}{2} \left( 1 + \cos(4\theta) \right) d\theta \right\} \left\{ \left[ \sin\theta \right]_{\theta = 0}^{\frac{3\pi}{8}} \right\} \]

\[ = \left\{ \left. \frac{1}{2} \left( \frac{1}{4} \sin(4\theta) + \sin(\theta) \right) \right|_{\theta = 0}^{\theta = \frac{3\pi}{8}} \right\} \]

\[ = \left\{ \frac{1}{8} \sin \left( \frac{3\pi}{2} \right) - \frac{1}{8} \sin(0) \right\} \left\{ \frac{3\pi}{8} - 0 \right\} \]

\[ = \left\{ \frac{1}{8} \cdot 1 - 0 \right\} \left\{ \frac{3\pi}{8} \right\} \]

\[ = \frac{3\pi}{64} \]

\[ \text{units}^2 \]
45/4

Find the length of the curve $0 \leq t \leq 10$

\[ x = 2 + t^3, \quad y = t^2 \]

\[ L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt \]

\[ = \int_0^{10} \sqrt{(3t^2)^2 + (2t)^2} \, dt \]

\[ = \int_0^{10} \sqrt{9t^4 + 4t^2} \, dt = \int_0^{10} t \sqrt{9t^2 + 4} \, dt \]

\[ \begin{align*}
  u &= 9t^2 + 4 & du &= 18t \, dt & \frac{du}{18} &= t \, dt \\
  t &= 0 \Rightarrow u = 4 & t &= 10 \Rightarrow u = 964 \\
  \int \frac{1}{18} \, du &= \frac{1}{18} \left[ \frac{u^{3/2}}{3/2} \right]_{u=4}^{u=964} = \frac{1}{27} (964^{3/2} - 4^{3/2}) \\
  &= \frac{1}{27} (964^{3/2} - 8) \text{ units}.
\end{align*} \]