**Problem 6**

Given the sequence \( a_n = \frac{(-1)^n n}{(n+1)^2} \), find the limit of the sequence as \( n \to \infty \).

**Solution**

\[ a_n = \frac{(-1)^n n}{(n+1)^2} \]

The sequence \( a_n \) converges to \(-\frac{2}{3}\).

**Problem 7**

Consider the series \( \sum_{n=1}^{\infty} \frac{2}{n^2+1} \).

**Solution**

This series converges by the integral test.

**Problem 8**

Use the integral test to determine whether the series \( \sum_{n=1}^{\infty} \frac{2}{n^2+1} \) converges.

**Solution**

The integral test shows that the series converges.

**Problem 9**

Consider the series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \).

**Solution**

This is a convergent p-series with \( p = 2 > 1 \).

**Problem 10**

Consider the series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \).

**Solution**

The series converges by the integral test.

**Problem 11**

Consider the series \( \sum_{n=1}^{\infty} \frac{1}{n} \).

**Solution**

This series diverges by the integral test.

**Problem 12**

Consider the series \( \sum_{n=1}^{\infty} \frac{1}{n} \).

**Solution**

This series diverges by the integral test.

**Problem 13**

Consider the series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \).

**Solution**

The series converges by the integral test.
4. Consider \( \frac{n+1}{n^2} = \frac{n+1}{n} \rightarrow 1 \) as \( n \rightarrow \infty \).

Since \( \frac{2+n}{n^2} \) diverges, it follows by the LCT that \( \sum_{n=1}^{\infty} \frac{2+n}{n^2} \) diverges.

5. Conv/Div. \( \sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n^2+1} \)

Solution: This is an Alternating Series. \( b_n = \frac{2n}{4n^2+1} \).

2. \( b_n > 0 \) and \( b_n > b_{n+1} \) for all \( n = 1, 2, 3, \ldots \).

\[ b_1 = \frac{2}{5}, \quad b_2 = \frac{4}{17}, \quad b_3 = \frac{6}{37}, \quad \text{etc.} \]

Or take derivative of \( f(x) = 2x/(4x^2+1) \).

3. \( \frac{2n}{4n^2+1} \rightarrow 0 \) as \( n \rightarrow \infty \).

By the AST \( \sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n^2+1} \) converges.

Bonus [CA]. [See Next Page]

C.B. 5 pts. \( 7428 \times 7 \). Solution: \( 7428 \times 7 = 71428 \)

Then \( 10000x = 74281428 \) and \( 9999x = 71421 \), so \( x = \frac{74281428 - 71421}{9999} \) which reduces to \( \frac{23804}{3333} \).