I. Reviewed problem (wrench) completed on yesterday's notes and done AFTER class.

II. We went over the TEST-DAY process for Calc 3.

III. Requested Problems:


The line of ∩ of T1: x + y + z = 1 and T2: x + z = 0.

So, \( n_1 = \langle 1, 1, 1 \rangle \) and \( n_2 = \langle 1, 0, 1 \rangle \)

2. \( \hat{V} = n_1 \times n_2 = \langle 1, 0, 1 \rangle \)

3. Find \( P \in T_1 \cap T_2 \).

Let \( x = 0 \), \( P_0 (0, 1, 0) \)

4. Param. \( x = a + t, y = 1, z = 0 - t \)

Symm \( \frac{x - 0}{1} = \frac{z}{-1} \)

B. §13.5: p. 867: #69 Show dist. between parallel planes

\( T_1: ax + by + cz + d_1 = 0 \) and \( T_2: ax + by + cz + d_2 = 0 \) is

\[ D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} \]

IV. Soln. I

1. Formula for dist. from a point to a plane.

\[ \hat{d} = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \]

2. Let \( P(x_1, y_1, z_1) \in T_1; ax + by + cz + d_1 = 0 \)

3. Consider the distance from \( T_1 \) to \( T_2 \) must be the same as the distance from \( P \) to \( T_2 \).

\[ D = \hat{d} = \frac{|ax_1 + by_1 + cz_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}} \]

4. \( a_1 + by_1 + cz_1 + d_2 = 0 \)

5. \( D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} \)

VI. Soln. II

1. I have a plane: (a) Find the line of intersection of \( T_1 \) and \( T_2 \).

(b) For 2 values of the parameter \( t \), find 2 points on \( L \).

(c) Use these two points and the given pt. \( P \) to determine a plane \( T \). Then \( T \) will contain \( L \) and \( P \).

2. Let \( \hat{V} = n_1 \times n_2 = \langle 1, 1, 1 \rangle \times \langle 2, 1, 3 \rangle = \langle 3 - 1, -2 - 3, 1 - 1 \rangle = \langle 2, -5, 0 \rangle \)

\( \hat{V} = \langle 2, -5, 0 \rangle \)

VII. After Class

Find an equation of the plane that passes through the point \( P(-1, 2, 1) \) and contains the line of intersection of the planes \( T_1: x + y - z = 2 \) and \( T_2: 2x - y + 3z = 1 \).

Soln. I

1. I have a plane: (a) Find the line of intersection of \( T_1 \) and \( T_2 \). Call this line \( L \).

(b) For 2 values of the parameter \( t \), find 2 points on \( L \).

(c) Use these two points and the given pt. \( P \) to determine a plane \( T \). Then \( T \) will contain \( L \) and \( P \).
3. Find \( P_0 \in L \) (let \( x = 0 \))
\[
\begin{align*}
\text{Let } x &= 0 \quad : \\
y - z &= 2 & \text{(from } T_1 \text{)} \\
-2 + \frac{3}{2}z &= 1 & \text{(not } T_2 \text{)} \\
\end{align*}
\]
\( P_0(0, \frac{3}{2}, \frac{3}{2}) \)

4. \( L : x = 0 + 2t, \quad y = \frac{3}{2} - 5t, \quad z = \frac{3}{2} - 3t \) 
\( y = \frac{3}{2} \)

5. \( t = 0 : P(0, \frac{3}{2}, \frac{3}{2}) \)
\( t = \frac{1}{2} \) : \( Q(1, 1, 0) \)

6. Now we have 3 points: \( Q(1, 1, 0), \ P_0(0, \frac{3}{2}, \frac{3}{2}) \) and \( P(-1, 1) \).

Let \( \overrightarrow{a} = \overrightarrow{P_0Q} = \langle -1, \frac{1}{2}, \frac{3}{2} \rangle \) and \( \overrightarrow{b} = \overrightarrow{PQ} = \langle -2, 1, 1 \rangle \)

Let \( \mathbf{n} = \overrightarrow{a} \times \overrightarrow{b} = \langle -1, \frac{1}{2}, \frac{3}{2} \rangle \times \langle -2, 1, 1 \rangle = \langle \frac{3}{2}, -3 + 3, 1 + 2 \rangle = \langle \frac{3}{2}, 0, 3 \rangle \) 
\( \mathbf{n} = \langle 1, -2, 3 \rangle \) This \( \mathbf{n} \) is the normal to \( T_1 \).

\( Q(1, 1, 0) \) is also on \( T_1 \).

7. Use \( a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \) for \( T_1 \).
\[
1(\bar{x} - 1) - 2(\bar{y} - 1) + 4(\bar{z} - 0) = 0 \quad \text{or } \quad \bar{x} - 2\bar{y} + 4\bar{z} + 1 = 0
\]

\( T_1: x - 2y + 4z + 1 = 0 \) in STD FORM.