I. Suppose

\[ w = x^2 + 3zt^2 \quad \text{and} \]

\[ x = 3u + v, \quad y = u - 2v, \quad z = 5u, \quad t = u^2 - v^2 \]

Find \[
\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial u} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial u} \]

\[
\frac{\partial y}{\partial u} = 2y + 3t^2 + 4zt + 2u \]

= \[3y^2 + 2yz + 5t^2 + 4ztu = \frac{\partial w}{\partial u}\]

See Fig 3, p. 970

II. Prob. 8.5.5: p. 974; # 8.

\[ z = \frac{x}{y}, \quad x = se^t \]

\[ y = 1 + se^{-t} \]

Find

\[ \frac{\partial x}{\partial s} \]

\[ \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} \]

\[ \frac{\partial z}{\partial s} = \frac{1}{y} e^t + \frac{-x}{y^2} e^t \]

\[ = \frac{ye^t - xe^{-t}}{y^2} \]

III. Implicit Diff., p. 972.

A. Example: The graph (in 2-space) of

\[ x^2 + y^2 = 4, \quad y = \pm \sqrt{4 - x^2} \]

OR

\[ f(x,y) = x^2 + y^2 - 4 \]

\[ f(x) = +\sqrt{4 - x^2} \quad \text{and} \quad f_y(x) = -\sqrt{4 - x^2} \]

II. Def. \[ F(x,y) = x^2 + y^2 - 4 \]

in (3-space)

Level Curve of \( F \) at 0 is \( F(x,y) = 0 \)

Graph is a circle in xy-plane.

Level Curve of \( F \) at 0 is \( F(x,y) = -4 \)

is a point!

Level Curve of \( F \) at \( z = -4 \) is \( F(x,y) = -4 \)

We could find \( f_y(x) \) and \( f_z(x) \) the way.

B. \[ \frac{\partial F}{\partial x} = 2x + 2y \frac{dy}{dx} - 0 \]

\[ \frac{\partial F}{\partial x} (F(x,y)) = \frac{\partial}{\partial x} (0) \]

4. \[ 2x + 2y \frac{dy}{dx} = 0 \]

5. \[ \frac{dy}{dx} = -\frac{x}{y} \]

6. \[ \frac{dx}{dy} = \frac{-x}{y} \quad \text{for example.} \]