Test #2 Answers on Web.

A. Recall $D_u f(P) = \nabla f(P) \cdot \hat{u}$, $P(x,y) \rightarrow P \rightarrow \hat{u} = \langle x, y \rangle$

B. Now $D_u f(P) = \nabla f(P) \cdot \hat{u} = \| \nabla f(P) \| \| \hat{u} \| \cos \theta$

Now $D_u f(P) = \| \nabla f(P) \| \cos \theta \leq \| \nabla f(P) \|$

The directional derivative can attain a given point $P$ of $f$.

And the min. val. of $f$ is $-\| \nabla f(P) \|$.

B. Find the direction in which max. rate of change occurs.

$\hat{u} = \frac{1}{\| \nabla f(3, 4) \|} \nabla f(3, 4) = \frac{1}{\sqrt{17}} \nabla f(3, 4)$

$\hat{u} = \left( \frac{-4}{\sqrt{17}}, \frac{3}{\sqrt{17}} \right)$

(And the answer in the back of the book is wrong! It says $\langle 1, 1 \rangle$.)

--- After Class ---

IV.

Problem 5.15.7: p. 398: #39 // Find eqs. of $T_{xy}$ and normal line to surface $\Sigma$ at $P(4, -1, 1)$.

$\Sigma: x^2 + 2y^2 + 3z^2 = 21.$

Solve (1) Check $P \in \Sigma: (4)^2 + 2(-1)^2 + 3(1)^2 = 16 + 2 + 3 = 21.$

2. $T_{xy}$ at $(x, y, z_0) (x - xo) + f_x(y, z_0) (y - yo) + f_z(xo, y, z_0) (z - z_0) = 0$

(eqs. 18.1.19; p. 984 - V, 1, P).

3. $f_x(x_0, y_0, z_0) = 2x_0, f_y(x_0, y_0, z_0) = 2y_0, f_z(x_0, y_0, z_0) = 2z_0$

And $f_x(x_0, y_0, z_0) = 8x_0, f_y(x_0, y_0, z_0) = 4y_0, f_z(x_0, y_0, z_0) = 6z_0$

4. $T_{xy}$ at $(x, y, z_0)$ $8(x - 4) - 4(y + 1) + 6(z - 1) = 0$

$8x - 4y + 6z - 42 = 0$

5. Normal line at $P(4, -1, 1)$

$\{ x = 4 + 8t \}$

$\{ y = -1 - 6t \}$

$\{ z = 1 + 6t \}$