Problem [§ 15.7: p. 998 : # 47] Find the dim. of a rect. box of max. vol. such that the sum of the lengths of its edges is a constant, c.

Facts of life. // Equations to be used.

Your job is to combine (correctly) & solve.

F.O.C.
@ V = 4wh
③ Sum of the lengths of the edges
L = 4h + 4w + 4h
L = 4(l + w + h)

④ F.O.P. @ Find \( \frac{dV}{dt} \) step.

\[ \frac{dV}{dt} = 4(l + w + h) = c \therefore l = \frac{c}{4} - w - h \]

- Figure this out for tomorrow. 5 pts. group/person.
- On 1 pt off everyone. Test.

Problem [§ 15.7: p. 998 : # 28] Find abs. max/min. in D.
\[ z = f(x,y) = 3 + xy - x - 2y \] D closed triangular region \( \cap \) vertices \( A(1,0) \), \( B(3,0) \), \( C(1,4) \).

Sol. 1. cont...

Sketch of Domain.

D

\[ z = f(x,y) = 3 + xy - x - 2y \]
\[ \frac{\partial z}{\partial x} = f_x(x,y) = y - 1 \]
\[ \frac{\partial z}{\partial y} = f_y(x,y) = x - 2 \]

\[ \frac{\partial^2 z}{\partial x^2} = f_{xx} (x,y) = 0 \]
\[ \frac{\partial^2 z}{\partial y^2} = f_{yy} (x,y) = 0 \]

\[ \frac{\partial^2 z}{\partial x \partial y} = f_{xy} (x,y) = 1 \]
and \( \frac{\partial^2 z}{\partial y \partial x} = f_{yx} (x,y) = 1 \)

\[ D(x,y) = f_{xx} (x,y) f_{yy} (x,y) - [ f_{xy} (x,y) ]^2 \]
\[ = - [1]^2 = -1 \]

\[ \frac{\partial z}{\partial x} = 0 \]
\[ \frac{\partial z}{\partial y} = 0 \]

\( x = 2 = 0 \Rightarrow (x = 2) \)

And \( \frac{\partial z}{\partial y} = 0 \)

\( x = 2 = 0 \Rightarrow (x = 2) \)

Therefore, the critical point is \((2,1)\).

\[ D(2,1) = -1 < 0 \]
and \( f_{xx} (2,1) = 0 \)

At \((2,1)\) the function has a saddle point.

( The remainder of this problem was done AFTER CLASS ).
3. On the line segment $\overline{AC}$, $z=1$, and so the restricted function \( \hat{f}(1,y) = 3 + y - 1 - 2y = 2 - y \) (which is a "line" w/ constant slope of $m=-1$)
So at the "A" end $\hat{f}(1,0) = 2$ and at the "C" end $\hat{f}(1,4) = -2$. Thus the max is achieved at $A(1,0)$ and the min is 2 at $C(1,4)$.
Analogously, the min of $-2$ is achieved at $C(1,4)$.

4. On the line segment $\overline{AB}$, $A(1,0)$, $B(5,0)$, $y=0$, so the restricted function $f(x,0) = 3 - z$. Thus, by similar considerations to those of step 3, we see that the values at the ends $f(1,0) = 2$ and $f(5,0) = -2$ are max and min, respectively.

5. On the segment $\overline{BC}$, we establish the eq of the segment in the $xy$ plane:
- $B(5,0), C(1,4)$
- $m = \frac{\Delta y}{\Delta x} = \frac{4}{1-5} = -1$,
- $y - y_1 = m(x-x_1)$
  \[ y - 0 = -1(x-5) \]
  \[ y = -x + 5 \]

This $f$ restricted to $y = 5-x$ is $\hat{f}(x,5-x) = 3 + x(5-x) - x - 2(5-x)$
\[ = 3 + 5x - x^2 - x - 10 + 2x = 7 + 6x - x^2 \]
and $\hat{f}'(x) = 6 - 2x \leq 0 \Rightarrow x = 3$ is a crit.
Also $y = 5-x$ \(. \) at $x = 3$, $y = 2$ and $\hat{f}(3,2) = 3 + 6 - 3 - 4 = 2$.

Thus $f(x,y) = 3 + xy - x - 2y$ has, over the closed region $D$, an abs. max. val. of $2$, achieved at $(1,0)$ and $(3,2)$ and an abs. min. val. of $-2$, achieved at $(5,0)$ and $(1,4)$.\[ \]