Interesting Concept from §17.2 which MAY be on Test #04. — center of mass of a wire! (p. 1100) — Look at it! (Example 3, p. 1101).

Not on Test — but on FINAL is §17.3
Fundamental Theorem of Line Integrals.

Recall FTC pt.II Says if \( f \) is a function which is integrable and if \( F \) is any anti-derivative of \( f \), then

\[
\int_{x=a}^{x=b} f(x) \, dx = F(b) - F(a)
\]

By way of analogy and/or comparison

If \( \vec{F} \) is a vector field which is conservative (so there exists a potential function, \( f \)) such that \( \nabla f = \vec{F} \), then

\[
\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))
\]

where \( a \leq \vec{r}(t) \leq b \) describes the path \( C \) and \( f \) is differentiable and \( \nabla f \) is continuous on \( C \).

Example: §17.3: p. 117: #3

Determine if \( \vec{F} \) is conservative, if it is, find a potential function i.e. \( f \) s.t. \( \nabla f = \vec{F} \).

\[ \vec{F}(x,y) = (6x+5y)\hat{i} + (5x+4y)\hat{j} \]

Solution:

\[ \vec{F}(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j} \]

Criterion for existence of a potential fun, \( f \) is

\[ \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \]

1. Check \( \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \).

\[ P(x,y) = 6x+5y \quad \frac{\partial P}{\partial y} = 5 \quad Q(x,y) = 5x+4y \quad \frac{\partial Q}{\partial x} = 5 \]

2. \( f(x,y) = \int P \, dx = \int (6x+5y) \, dx = 3x^2 + 5xy + g(y) \)

3. \( Q(x,y) = \frac{\partial f}{\partial y} \quad \text{MUST} \)

\[ 5x+4y = 5x + g'(y) \Rightarrow g'(y) = 4y \Rightarrow g(y) = 2y^2 + C \]

4. \( f(x,y) = 3x^2 + 5xy + 2y^2 + C \) is the potential function.