II. Intro. (went over web page).

III. Section 11.1 (p. 822). Sequences & Summation Notation

A. Definition: A sequence is a list (of numbers) in a certain order.

- Finite sequence (has beginning and ending):
  - Example: \(2, 4, 6\) (definite order)
  - or \(\{2, 4, 6\}\)

- Infinite sequence
  - Example: \(1, 3, 9, \ldots\)

  "See pattern" \(a_n = 3^{n-1}\)

N.B. Notice \(a_n = 3^{n-1}\)

- \(n=1\) the first element is \(a_1 = 3^{1-1} = 1\)
- \(n=2\) "Second" \(a_2 = 3^{2-1} = 3\)

What is the 15th term (element) of the seq?
- \(n=15\) \(a_{15} = 3^{14} = 4,782,969\)

So, sequences have to do with patterns.

B. Problems (p. 830)

Find the first 4 terms and the 100th term.

1. \(a_n = n + 1\)
   - \(a_1 = 1 + 1 = 2\)
   - \(a_2 = 2 + 1 = 3\)
   - \(a_3 = 3 + 1 = 4\)
   - \(a_4 = 4 + 1 = 5\)
   - \(a_{100} = 100 + 1 = 101\)

2. 2, 3, 4, 5 and \(a_{100} = 101\).
   - The first 4 terms are \(2, 3, 4, 5\).

3. \(a_n = \frac{1}{n+1}\)
   - \(a_1 = \frac{1}{1+1} = \frac{1}{2}\)
   - \(a_2 = \frac{1}{2+1} = \frac{1}{3}\)
   - \(a_3 = \frac{1}{3+1} = \frac{1}{4}\)
   - \(a_4 = \frac{1}{4+1} = \frac{1}{5}\)
   - \(a_{100} = \frac{1}{101}\)

4. 2, 3, 4, 5 and \(a_{100} = 101\).
   - The first 4 terms are \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\) and the 100th term is \(\frac{1}{101}\).

5. \(a_n = \frac{(-1)^n}{n^2}\)
   - \(a_1 = \frac{-1}{1^2} = -1\)
   - \(a_2 = \frac{1}{4}\)
   - \(a_3 = \frac{-1}{9}\)
   - \(a_4 = \frac{1}{16}\)
   - \(a_{100} = \frac{-1}{10,000}\)
The first 4 terms are \(-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}\), and the 100th term is \(\frac{1}{10000}\).

Recursively Defined Sequences.

Example, Recursive Formula (Pattern)

\[ a_n = a_{n-1} + 3 \]

and \(a_1 = -5\)

List the first 5 terms.

Solution

\[ a_1 = -5 \]
\[ a_2 = a_1 + 3 = -5 + 3 = -2 \]
\[ a_3 = -2 + 3 = 1 \]
\[ a_4 = 1 + 3 = 4 \]
\[ a_5 = 4 + 3 = 7 \]

The first 5 terms are \(-5, -2, 1, 4, 7\).