III. We went over the problem that we started in class on Tue which I finished after class — It is on yesterday’s notes.

Be sure to read, study, and understand
Ex 1 (p. 857), Ex 2 (p. 857), Ex 3 (p. 858)

Prove that \( n < 2^n \) for all natural numbers \( n \).
Proof: \( \square \) Verify PO: \( 1 < 2^1 = 2 \)

II. Thus we have shown by the PMI that \( n < 2^n \) for all natural numbers \( n \).

S 11.6 THE BINOMIAL THEOREM (p. 860)

A. Pascal’s Triangle
\[
\begin{array}{ccccccc}
0 & & & & & & 1 \\
1 & & & & & 1 & 1 \\
1 & 4 & 6 & 4 & 1 & 1 \\
5 & 10 & 15 & 20 & 15 & 6 & 1 \\
\end{array}
\]

B. Uses.
\[
(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \cdots + \binom{n}{n} x^0 y^n
\]

Proof:
\[
(x+y)^2 = (x+y)(x+y) = (x+y)^2 + 2xy
\]

\[
(x+y)^3 = (x+y)^2 = x^2 + 2xy + y^2
\]

C. Example:
\[
(x+y)^6 = (x+y)^2 \cdot (x+y)^2 \cdot (x+y)^2
\]

\[
(x+y)^6 = (x+y)^2 + (x+y)^2 + (x+y)^2
\]

\[
(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6
\]