Intermediate Value Theorem for Polynomials, p. 255.

1) \( P \) is a poly. fun. and
2) \( P(a) \) and \( P(b) \) have opposite signs

(Suppose \( a < b \) for purposes of discussion)

\( (a, P(a)) \)

\( (b, P(b)) \)

There exists at least one number \( c \) (on the \( x \)-axis) between \( a \) and \( b \) \( (a < c < b) \) such that (with the property that) \( P(c) = 0 \).

§ 3.2 Division of Polys, p. 265,

A) Long Division (Numbers)

\[
\begin{array}{c|ccccc}
 & 789 & 4 & 246 & 326 & 22 \\
\hline
32 & 0 & 246 & 326 & 22 \\
-149 & -128 & 214 & 192 & 22 \\
-128 & -128 & 120 & 0 \\
\hline
22 & 22 & 0 \\
\end{array}
\]

\[
\frac{7894}{32} = 246 + \frac{22}{32}
\]

\[
\frac{128}{32} = 4
\]

B) Long Division of Polynomials.

\[
\frac{5x^2 - 2x + 2}{x^2 + x - 2} = \frac{5x^4 + 3x^3 - 10x^2 + x + 1}{x^4 + 3x^3 - 10x^2 + x + 1}
\]

\[
\frac{5x^4 + 3x^3 - 10x^2 + x + 1}{x^2 + x - 2} = 5x^2 - 2x + 2 + \frac{-5x + 5}{x^2 + x - 2}
\]

\[
\frac{5x^4 + 3x^3 - 10x^2 + x + 1}{x^2 + x - 2} = 5x^2 - 2x + 2 - \frac{5x - 5}{x^2 + x - 2}
\]

C) If the degree of the divisor is 1, we can use synthetic division.

\[
\frac{5x^3 + 2x^2 - 3x + 7}{x + 4} \quad \text{OLD WAY} \quad \frac{5x^2 - 18x + 69}{x + 4}
\]

\[
= 5x^2 - 18x + 69 - \frac{269}{x + 4}
\]

\[
\frac{18x^2 + 2x + 2}{x^2 + x - 2}
\]

\[
= \frac{189x + 7}{x^2 + x - 2}
\]

\[
= \frac{189x + 7}{x^2 + x - 2}
\]

\[
= \frac{269}{x + 4}
\]

\[
= \frac{269}{x + 4}
\]
SD, Synthetic Division

\[ \begin{array}{c|cccc}
-4 & 5 & 2 & -3 & 7 \\
\hline
 & -20 & 72 & -276 \\
\end{array} \]

Remainder: 269

\[ P(x) = 5x^3 + 2x^2 - 3x + 7 \]

\[ \frac{P(-4)}{x+4} = \frac{-269}{x+4} \]

P(-4) = 269

Also SD tells me an "easy" way to evaluate a polynomial function at a number.