
Factor $P(x) = x^4 + 7x^3 + 16x^2 + 15x + 9$

"over the reals," (into the product of linear factors and irreducible quadratic factors).

Solve for real zeros. $\pm 1, \pm 3, \pm 9$

Try (-1): $P(-1) = (-1)^4 + 7(-1)^3 + 16(-1)^2 + 15(-1) + 9$

$= 1 - 7 + 16 - 15 + 9 = 6 
eq 0$

Try (-3): $P(-3) = (-3)^4 + 7(-3)^3 + 16(-3)^2 + 15(-3) + 9$

$= 81 - 189 + 144 - 45 + 9 = 0$

This is a factor

$P(x) = (x+3)^2(x^2 + x + 1)$

Factor the same poly. fun. "over the complex numbers."

We already have $P(x) = (x+3)^2(x^2 + x + 1)$

It remains to factor $x^2 + x + 1$.

I'll do it 2 ways...

Method GR Set $x^2 + x + 1 = 0$

Method Graphical Setting $x^2 + x + 1 = 0$

$x^2 + x + \frac{1}{4} = \frac{3}{4}$

$x + \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$

$x = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$
II Q7 #4: \[ \frac{40 - 35}{2+i} \quad \frac{A - C}{B - D} = \frac{AD - BC}{BD} \]

\[ \frac{40}{2-i} - \frac{35}{2-i} = \frac{40(2+i) - 35(2+i)}{(2-i)(2+i)} \]
\[ = \frac{80 - 40i - 70 - 35i}{4 + 1} = \frac{10 - 75i}{5} = 2 - 15i \]

\[ (A+B)(A-B) = A^2 - B^2 \]

II Q7 #2(c) \[ x^4 - 5x^2 - 6 \] Factor completely.

\[ A^2 - 5A - 6 = (A - 6)(A + 1) \]

\[ \therefore \quad P(x) = (x^2 - 5x^2 - 6) = (x^2 - 6)(x^2 + 1) \] (***)

2 \[ x^2 - 6 = (x + \sqrt{6})(x - \sqrt{6}) \]

3 \[ x^2 + 1 = 0 \]
\[ x = \pm i \]
\[ x = \pm \sqrt{-1} \]
\[ x = \pm i \]

\[ x = i \] or \[ x = -i \]

4 \[ \therefore \quad P(x) = (x + i)(x - i)(x - i)(x + i) \]

There are 2 complex roots!

IV Construct a poly: \( 4^{th} \) degree w/ zeros \( 2 - i, 3, -1 \) & real coefficients.

Class ends

After Class: Finish this problem.

Note: If \( 2 - i \) is a zero of \( P(x) \), then \( 2 + i \) (the complex conjugate) must also be a zero of \( P(x) \).

See the "Conjugate Zeros Theorem" in Section 3.8, p. 296.

Thus the 4 zeros of \( P(x) \) are \( 2 - i, 2 + i, 3, -1 \).

3 Now the question is: "How do we get this into STD. FORM?"

I'll break it down into two problems, then I'll put together my answers:

(a) \( P_1(x) = (x - 3)(x + 1) \)
\[ = (x - 3)(x + 1) \]
\[ = (x^2 - 2x - 3) \]

(b) \( P_2(x) = (x - (2 - i))(x - (2 + i)) \)
\[ = (x - 2 + i)(x - 2 - i) \]
\[ = (x - 2)^2 - (i)^2 \]
\[ = x^2 - 4x + 4 + 1 \]
\[ = x^2 - 4x + 5 \]

(c) So \( P(x) = P_1(x)P_2(x) = (x^2 - 2x - 3)(x^2 - 4x + 5) \)
\[ = x^4 - 4x^3 + 5x^2 - 2x^2 + 8x - 10x - 3x^2 + 12x - 15 \]
\[ = x^4 - 6x^3 + 10x^2 + 2x - 15 \] Has zeros \( 2 - i, 3, -1 \).