
Show $\cos \alpha = \frac{a}{\| \vec{v} \|}$ when $\vec{v} = \langle a, b, c \rangle$

III WBB: Given the line $\mathcal{L}$ in param form

$x = 2 + 3t, \ y = 1 - t, \ z = 5t,$

Find the parametric equations for a line $\mathcal{L}_1$ parallel to $\mathcal{L}$ and which passes through the point $(1, 2, 3)$.

Ans: $x = 1 + 3t, \ y = 2 - t, \ z = 3 + 5t$

III §12.5: p. 872: # 57: Find param. eq. for line of intersection of

$\Pi_1: x + y + z = 1$ \& $\Pi_2: x + y = 2$.

Soln: 1 $\vec{n}_1 = \langle 1, 1, 1 \rangle$ \& $\vec{n}_2 = \langle 1, 1, 0 \rangle$

2 Let $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = (a - 1) \hat{z} - (c - 1) \hat{j} + (c - 1) \hat{k}$
\[ \vec{v} = -\hat{i} + \hat{j} = \langle -1, 1, 0 \rangle \]

3. Find a point \( P_0 \in \Pi_1 \cap \Pi_2 \)

\( \Pi_1: x + y + z = 1 \)
\( \Pi_2: x + y = 2 \)

Let \( (x = 0) \), then \( y + z = 1 \) and \( y = 2 \)

from \( \Pi_1 \)

from \( \Pi_2 \)

\[ \therefore z = -1 \]

\[ \therefore P_0(0, 2, -1) \text{ is in both planes.} \]

4. The line of intersection of \( \Pi_1 \parallel \Pi_2 \) is

\[ \mathcal{L}: x = 0 - t, \ y = 2 + t, \ z = -1. \]

IV. Example: §12.5: p.872: #53. Find the point where the line meets the plane.

\( \mathcal{L}: x = 1 - t, \ y = 3t, \ z = 1 + t; \ \Pi: 2x - y + 3z = 6 \)

(\( P_0 \in \mathcal{L} \cap \Pi \))

Solution 1: Subs \( \mathcal{L} \) into \( \Pi \):

\[ 2(1-t) - (3t) + 3(1+t) = 6 \]

2. And solve for \( t \):

\[ 2 - 2t - 3t + 3 + 3t = 6 \]

\[ -2t = 1 \quad \Rightarrow t = -\frac{1}{2} \]

3. Subs \( t = -\frac{1}{2} \) into \( \mathcal{L} \)

\[ x = 1 + \frac{1}{2}, \ y = -\frac{3}{2}, \ z = 1 - \frac{1}{2} \]

\[ P_0\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right) \]
Start: § 12.6: Cylinders & Quadric Surfaces (p. 873)

A. Cylinder

Generating curve (Generator)

"moving" line — directrix

right parabolic cylinder.

B. Quadric Surfaces. (p. 875) — Graph of THIS eq.

\[ Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Ix + K = 0 \]

For instance If \( A, B, C = 1 \) and \( D = E = F = G = H = I = 0 \) and \( K < 0 \), then the graph is a sphere

1. So a sphere is a quadric surface.

What other Q.S.'s do we need to know?

2. Ellipsoids (p. 875)

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]