\[ \vec{r}(t) = [(v_0 \cos \alpha) \hat{i} t + (v_0 \sin \alpha) t - \frac{1}{2} gt^2] \hat{j} \]

**Ideal Projectile Motion Equation (Model).**

**Extra Equations. (p. 907)**

- **Max Ht.**
  \[ y_{\text{max}} = \frac{(v_0 \sin \alpha)^2}{2g} \]

  Take the vertical component of position
  \[ y(t) = (v_0 \sin \alpha) t - \frac{1}{2} gt^2 \]
  Take \( y'(t) = v_0 \sin \alpha - gt \) set \( t = t_{\text{max}} \) and solve
  \[ t_{\text{max}} = \frac{v_0 \sin \alpha}{g} \]

  So
  \[ y(t_{\text{max}}) = \frac{1}{2} g \left( \frac{v_0 \sin \alpha}{g} \right)^2 - \frac{1}{2} g \left( \frac{v_0 \sin \alpha}{g} \right)^2 = \frac{(v_0 \sin \alpha)^2}{2g} - \frac{(v_0 \sin \alpha)^2}{2g} = \frac{(v_0 \sin \alpha)^2}{2g} \]
2. How far it goes?

\[ \vec{r}(t) = (v_0 \cos \alpha) t \hat{i} + (v_0 \sin \alpha) t - \frac{1}{2} gt^2 \hat{j} \]

"Far" = "x" I want to know the value(s) of x when y = 0

Find \( t_{\text{max}} \) by setting \( y = 0 \)

\[(v_0 \sin \alpha) t - \frac{1}{2} gt^2 = 0\]

\[t \left[ v_0 \sin \alpha - \frac{1}{2} gt \right] = 0\]

\[\therefore \text{ Either } (t = 0) \text{ or } \left( v_0 \sin \alpha - \frac{1}{2} gt = 0 \right) \]

\[\therefore \left( t = \frac{2v_0 \sin \alpha}{g} \right) \]

\[\therefore \left( x = \frac{v_0 \cos \alpha}{g} \right) \]

\[\therefore \left( x = \sin (2\alpha) \frac{v_0^2}{g} \right) \]

I'll put some MML-HW on line.