II. Do Homework.

§ 14.3 Partial Derivatives (p. 968).

A. Derivatives of functions of 1 variable.

If \( y = f(x) \), then the derivative of \( f \) at \( "a" \)

is defined by

\[
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

or

\[
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
\]

**IF** the limit exists.

In more generality, a function \( y = f(x) \) is
differentiable on an interval iff

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ exists } \forall x \in I \text{ (the interval)}
\]

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}
\]

B. If \( z = f(x,y) \) on some Domain D. Then
we define "the partial derivative of \( f \) with
respect to \( x \) as

\[
\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}
\]

if \( \lim \) exists.
Similarly, 
\[
\frac{df}{dy} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}
\]

\[\square\text{ Some Examples.}\]

1. Find \( \frac{d}{dx} f(x,y) \) where \( f(x,y) = x^2y^3 \).
   Solution — Fortunately we have RULES of PARTIAL DERIVATIVES just as we have RULES for "Regular" Derivatives
   \[
   \frac{d}{dx} (x^2y^3) = 2xy^3
   \]

2. \( \frac{d}{dy} (x^2y^3) = 3x^2y^2 \)

3. \( \frac{d}{dx} (\sin(xy) \cos(x^2y)) = -\sin(xy) \sin(x^2y) \cdot 2xy \\
   + \cos(x^2y) \cdot \cos(xy) \cdot y \\
   = y \cos(xy) \cos(x^2y) - 2xy \sin(xy) \sin(x^2y) \)

4. Evaluate \( \frac{df}{dy} @ (1,2) \) if \( f(x,y) = x^2 + 3xy + y^2 \).
   Solution: \( \frac{d}{dy} (x^2+3xy+y^2) = 3x + 2y \)
   \( \frac{df}{dy} \bigg|_{(1,2)} = 3(1) + 2(2) = 7 \)