A volume w/ Double Integrals.

Find the volume defined in 3-space in the 1st Octant by \( x = 3 \) and \( z = 8 - 2y \).

**Solution**

1. First Graph the Region \( R \) in the context

\[
V = \iiint_{R} (8 - 2y) \, dA
\]

Solve the "system"

\[
\begin{align*}
\text{S} & = 8 - 2y \\
\text{O} & = 8 - 2y \quad \therefore y = 4
\end{align*}
\]

2. \( V = \iiint_{R} (8 - 2y) \, dA = \int_{x=0}^{x=3} \int_{y=0}^{y=4} (8 - 2y) \, dy \, dx \\

= \int_{x=0}^{x=3} \left[ 8y - y^2 \right]_{y=0}^{y=4} \, dx = \int_{x=0}^{x=3} (32 - 16) \, dx \\

= \int_{x=0}^{x=3} 16 \, dx = 48 \text{ units}^3
\]

3. **Summary**: \( V = \iiint_{R} (8 - 2y) \, dA = 48 \text{ units}^3 \).

4. Note: If my only requirement is to solve this problem, here is an alternative:

Area of End "Face" = 16 units$^2$

"Thickness" of the solid = 3 units

Volume = 16·3 = 48 units$^3$
§ 15.3: Double Integrals In Polar Form (P. 1076)

A. Sketch

1. We want the coordinates \((r, \theta)\) of \(P_1\) and \(P_2\).

\[ P_1(r_1, \theta_1) \quad r\text{-coordinate} \quad r_k \text{ is halfway between } r_1 \text{ and } r_2 \]

\[ P_2(r_2, \theta_2) \]

\[ r_k = r_1 + \frac{Dr}{2} \quad r_k = r_1 + \frac{Dr_k}{2} \quad \text{If all the } \Delta r_k \text{'s are the same, just call it } \Delta r. \]

\[ r_1 = r_k - \frac{Dr}{2} \]

\[ r_2 = r_k + \frac{Dr}{2} \]

and similarly

2. What is the formula for the area of a circular sector?

\[ A_S = \frac{\theta}{2} r^2 \]
### Area of Polar Rectangle

The area of the "polar rectangle" can be calculated as the difference of two circular sectors: the "Big" one minus the "Small" one.

\[
A_{\Delta A_k} = \text{Area Big Sector} - \text{Area Smaller Sector}.
\]

\[
= \frac{\Delta \theta}{2} r_2^2 - \frac{\Delta \theta}{2} r_1^2
\]

(Now we play with it.)

\[
= (r_2^2 - r_1^2) \frac{\Delta \theta}{2}
\]

\[
= (r_2^2 + r_1^2)(r_2 - r_1) \frac{\Delta \theta}{2}
\]

\[
= [(r_k + \Delta r) + (r_k - \Delta r)] [(r_k + \frac{\Delta r}{2}) - (r_k - \frac{\Delta r}{2})] \frac{\Delta \theta}{2}
\]

\[
= (2r_k)(2\frac{\Delta r}{2}) \frac{\Delta \theta}{2} = r_k \Delta r \Delta \theta
\]

\[
\therefore A_{\Delta A_k} = r_k \Delta r \Delta \theta
\]

And actually in our book they use $\Delta A_k$ to designate the area of a polar rectangle, so our formula should read:

\[
\Delta A_k = r_k \Delta r \Delta \theta
\]

But what does this MEAN? Since, roughly, the $\Delta \theta$ is the width of the polar rectangle and $\Delta r$ is the height of
the polar rectangle, you might (incorrectly) think that the area would be $\Delta r \Delta \theta \left[ \text{Similar to the rectangular case where } A = LW \right]$. But here you see there is a difference — the farther away from the origin (pole) that a "polar rectangle" is, the larger in area it becomes, due to the factor $r_k$.

Keep this in mind with all polar integration.