\[ 16.2: \text{(Related Applications of the Line Integral)} \ (P.1133) \hspace{1em} \text{Could be Titled} \]

\[ \text{VECTOR FIELDS, WORK, CIRCULATION, FLUX.} \]

\[ \text{A Vector Field: } \vec{F}(x,y,z) = M(x,y,z) \hat{i} + N(x,y,z) \hat{j} + P(x,y,z) \hat{k} \]

A vector field \( \vec{F} \) is said to be **smooth** if

\[ M, N, P \text{ are smooth (and...)} \]

Continous if \( M, N, P \) are continuous;

Differentiable if \( M, N, P \) are differentiable.

\[ (P.1136) \]

\[ \text{Gradient Field} \quad \text{A gradient field is a vector field such that } \vec{F} = f(x,y,z) \text{ s.t. } \nabla f(x,y,z) = f_x(x,y,z) \hat{i} + f_y(x,y,z) \hat{j} + f_z(x,y,z) \hat{k} \]

is equal to \( \vec{F} \); i.e. \( \vec{F} = \nabla f \)

This function \( f \) which is related to \( \vec{F} \) is called the potential function.

\[ \text{C Work. Defn: } W = \int_{t=a}^{t=b} \vec{F} \cdot \vec{T} \, ds \text{ where } \]

\[ \vec{F} = M\hat{i} + N\hat{j} + P\hat{k} \text{ over a smooth curve } \]

\[ \vec{F}(t) = f(t) \hat{i} + g(t) \hat{j} + h(t) \hat{k} \quad a \leq t \leq b. \]

\[ (P.1138) \]

\[ \text{Six Ways to Work! Tbl. 16.2.} \]

\[ \boxed{W = \int_{t=a}^{t=b} \vec{F} \cdot \vec{T} \, ds \quad \text{Defn:}} \]

(My Book has a mistake here! It lists \( W \) in bold, as if it were a vector.)
\[ \text{Compact Form.} \]

\[ \int_{t=a}^{t=b} \vec{F} \cdot d\vec{r} \]

"dt" in the mix.

\[ \int_{t=a}^{t=b} \vec{F} \cdot \frac{d\vec{r}}{dt} \, dt \]

\[ \int_{t=a}^{t=b} (M \frac{dg}{dt} + N \frac{dh}{dt} + P \frac{dk}{dt}) \, dt \quad \vec{F} = \langle g, h, k \rangle \]

The "integrand" is called a differential FORM.

\[ \int_{t=a}^{t=b} M \, dx + N \, dy + P \, dz \]

Tomorrow we shall Work on Work!